UW-PARKSIDE

INTERMEDIATE ALGEBRA SKILLS MANUAL

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To my mother

Welcome to UW-Parkside's Intermediate Algebra Skills Review!

This manual is intended to provide you with meaningful practice materials pointedly focused upon Intermediate Algebra problem types and techniques essential for success in introductory college-credit-bearing mathematics courses. This foundational content has been divided into 35 distinct sections for which on-demand online video lectures have been recorded, and each of these 35 videos is subsequently tied to a corresponding pair of exercise sets in this manual labeled EXAMPLES and PRACTICE. While the videos represent your most supported exposure to the material since they are accompanied by verbal explanations and written procedures, each of the two corresponding exercise sets removes some of that initial support as follows:

> EXAMPLES: Solutions to EXAMPLES exercises are fully written out, so although the verbal explanations from the corresponding video lectures are not present, a significant amount of support remains. As you attempt to solve problems from EXAMPLES sets on your own, therefore, you can refer to the written work provided to diagnose errors if you have difficulties.

> PRACTICE: Only final answers to PRACTICE problems are provided, so the written component from the EXAMPLES sets is largely absent. You will therefore have to do the work independently to arrive at a final answer to a PRACTICE exercise, and when your final answer matches the one recorded on the PRACTICE key, you can be confident that you have completed that exercise correctly.

It is my hope that viewing the content videos and working through the associated EXAMPLES and PRACTICE sets presented in this manual will help you to refresh the mathematics you already know, formulate questions in the areas you wish to review more deeply, and build your confidence as you engage the material in your introductory college-credit-bearing mathematics course. Thank you for your engagement with these Intermediate Algebra Skills Review resources, and I am confident that the work you do with them will leave you well positioned to excel in your first college-level mathematics course.

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Richard W. Karwatka

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CHAPTER I

LINEAR EQUATIONS AND INEQUALITIES

Simplify.

1. $\frac{3}{4}x^2 + 2x - \frac{1}{3} - \frac{1}{6}x^2 + \frac{5}{2}x + 1$

$= \boxed{\frac{7}{12}x^2 + \frac{9}{2}x + 3}$

x^2	x	Constant
$\frac{3}{4} - \frac{1}{6}$	$2 + \frac{5}{2}$	$2 + 1$
$\frac{3 \cdot 3}{4 \cdot 3} - \frac{1 \cdot 2}{6 \cdot 2}$	$\frac{2 \cdot 2}{1 \cdot 2} + \frac{5}{2}$	3
$\frac{9}{12} - \frac{2}{12}$	$\frac{4}{2} + \frac{5}{2}$	
$\frac{7}{12}$	$\frac{9}{2}$	

2. $-1.5(3x - 8y - 5)$

$= \boxed{-4.5x + 12y + 7.5}$

$\begin{array}{r} 1.5 \\ \times\ 3 \\ \hline 4.5 \end{array}$ $\qquad \begin{array}{r} 1.5 \\ \times\ 8 \\ \hline 12.0 \end{array}$ $\qquad \begin{array}{r} 1.5 \\ \times\ 5 \\ \hline 7.5 \end{array}$

3. $2\left(7x^2 - 9x + 1\right) - 5\left(3x^2 + 4x - 6\right)$

$= 14x^2 - 18x + 2 - 15x^2 - 20x + 30$

$= \boxed{-x^2 - 38x + 32}$

4. $0.35\left(6x^2 - 7xy\right) + 0.4\left(2x^2 + 5xy - 9y^2\right)$

$= 2.1x^2 - 2.45xy + 0.8x^2 + 2xy - 3.6y^2$

$= \boxed{2.9x^2 - 0.45xy - 2.8y^2}$

$$\begin{array}{c} \overset{3}{.35} \\ \underline{\times\ 6} \\ 2\overset{\frown}{10} \end{array} \qquad \begin{array}{c} \overset{3}{.35} \\ \underline{\times\ 7} \\ 2\overset{\frown}{.45} \end{array}$$

x^2	xy	y^2
$2.1 + 0.8$	$-2.45 + 2$	$0.8 - 3.6$
2.9	-0.45	-2.8

5. $3\left(8x^2 + 2x - 5\right) + 7\left(4x - 1\right) - 10\left(9x^2 + x - 7\right)$

$= 24x^2 + 6x - 15 + 28x - 7 - 90x^2 - 10x + 70$

$= \boxed{-66x^2 + 24x + 48}$

$\overset{5}{\tfrac{1}{2}} \cdot \overset{5}{\cancel{10}} = 5 \qquad \tfrac{1}{2} \cdot 3 = \tfrac{3}{2} \qquad \tfrac{1}{2} \cdot 2 = 1 \qquad 4 \cdot \tfrac{2}{3} = \tfrac{8}{3} \qquad \tfrac{2}{8} \cdot \cancel{18}^{3} = 6 \qquad \tfrac{2}{8} \cdot \overset{4}{\cancel{20}} = 8 \qquad \tfrac{2}{5} \cdot 3 = \tfrac{6}{5}$

6. $\tfrac{1}{2}\left(10x^2 - 3xy + 2y^2\right) - 4\left(2x^2 + \tfrac{2}{3}xy - 5y^2\right) + \tfrac{2}{5}\left(15x^2 - 20xy - 3y^2\right)$

$= 5x^2 - \tfrac{3}{2}xy + y^2 - 8x^2 - \tfrac{8}{3}xy + 20y^2 + 6x^2 - 8xy - \tfrac{6}{5}y^2$

$= \boxed{3x^2 - \tfrac{73}{6}xy + \tfrac{99}{5}y^2}$

$\dfrac{x^2}{5 - 8 + 6}$

3

$\dfrac{xy}{-\tfrac{3}{2} - \tfrac{8}{3} - 8}$

$-\tfrac{3 \cdot 3}{2 \cdot 3} - \tfrac{8 \cdot 2}{3 \cdot 2} - \tfrac{8 \cdot 6}{1 \cdot 6}$

$-\tfrac{9}{6} - \tfrac{16}{6} - \tfrac{48}{6}$

$-\tfrac{73}{6}$

$\dfrac{y^2}{1 + 20 - \tfrac{6}{5}}$

$\tfrac{21 \cdot 5}{1 \cdot 5} - \tfrac{6}{5}$

$\tfrac{105}{5} - \tfrac{6}{5} = \tfrac{99}{5}$

Simplify.

1. $\frac{5}{6}x^2 - 4x + \frac{3}{2} - \frac{2}{9}x^2 + \frac{7}{3}x - 1$

$$\frac{11}{18}x^2 - \frac{5}{3}x + \frac{1}{2}$$

2. $-2.4(9x - 5y + 7)$

$$-21.6x + 12y - 16.8$$

3. $3\left(7x^2 - 8x + 4\right) - 8\left(3x^2 + 5x - 6\right)$

$$-3x^2 - 64x + 60$$

4. $0.65 \left(8x^2 - 3xy\right) + 0.2 \left(5x^2 - 9xy + 11y^2\right)$

$6.2x^2 - 3.75xy + 2.2y^2$

5. $7 \left(6x^2 - 3x + 4\right) + 2 \left(5x - 8\right) - 5 \left(8x^2 + x - 3\right)$

$2x^2 - 16x + 27$

6. $\frac{1}{4} \left(20x^2 - 2xy + 5y^2\right) - 3 \left(15x^2 + \frac{2}{9}xy - y^2\right) + \frac{5}{8} \left(16x^2 - 40xy - 2y^2\right)$

$-30x^2 - \frac{157}{6}xy + 3y^2$

Solve.

1. $2x - 7 = -10x - 37$

$+10x +7 \quad +10x +7$

$\dfrac{12x}{12} = \dfrac{-30}{12}$

$\boxed{x = -\dfrac{5}{2}}$

2. $\overset{100}{\overbrace{(0.16x - 0.05)}} = \overset{100}{\overbrace{(0.01 - 0.08x)}}$

$16x - 5 = 1 - 8x$

$+8x +5 \quad +5 +8x$

$\dfrac{24x}{24} = \dfrac{6}{24}$

$\boxed{x = \dfrac{1}{4}}$

3. $\overset{6}{\overbrace{\left(\tfrac{2}{3}x + 4\right)}} = \overset{6}{\overbrace{\left(\tfrac{7}{6}x + 7\right)}}$

$4x + 24 = 7x + 42$

$-4x -42 \quad -4x -42$

$\dfrac{-18}{3} = \dfrac{3x}{3}$

$\boxed{-6 = x}$

4. $12x + 5 = 16x - 7$

$-12x + 7 \quad -12x + 7$

$\dfrac{12}{4} = \dfrac{4x}{4}$

$\boxed{3 = x}$

5. $\overset{10}{\overbrace{(0.7x + 3)}} = \overset{10}{\overbrace{(1.3 - x)}}$

$7x + 30 = 13 - 10x$

$+10x - 30 \quad -30 + 10x$

$\dfrac{17x}{17} = \dfrac{-17}{17}$

$\boxed{x = -1}$

6. $\overset{30}{\overbrace{\left(\frac{3}{2}x - 1\right)}} = \overset{30}{\overbrace{\left(\frac{7}{10}x - \frac{7}{15}\right)}}$

$45x - 30 = 21x - 14$

$-21x + 30 \quad -21x + 30$

$\dfrac{24x}{24} = \dfrac{16}{24}$

$\boxed{x = \frac{2}{3}}$

Solve.

1. $3x - 8 = -12x - 43$

$x = -\frac{7}{3}$

2. $0.27x - 0.04 = 0.01 - 0.18x$

$x = \frac{1}{9}$

3. $\frac{5}{4}x + 2 = \frac{3}{8}x - 5$

$x = -8$

4. $17x - 20 = 11x + 10$

$x = 5$

5. $0.4x + 9 = 4.8 - x$

$x = -3$

6. $\frac{2}{3}x + 6 = \frac{7}{5}x + \frac{27}{10}$

$x = \frac{9}{2}$

Solve.

1. $3(2x - 5) - (4x + 3) = 5(1 - x) + 26$

$$6x - 15 - 4x - 3 = 5 - 5x + 26$$
$$2x - 18 = -5x + 31$$
$$+5x + 18 \quad +5x + 18$$
$$\frac{7x}{7} = \frac{49}{7}$$
$$\boxed{x = 7}$$

2. $\overset{56}{\left(\dfrac{5 - 2x}{8} - \dfrac{x + 12}{14}\right)} = \overset{56}{\left(\dfrac{5x - 1}{7} - \dfrac{3x + 5}{56}\right)}$

$$7(5 - 2x) - 4(x + 12) = 8(5x - 1) - 1(3x + 5)$$
$$35 - 14x - 4x - 48 = 40x - 8 - 3x - 5$$
$$-18x - 13 = 37x - 13$$
$$-37x + 13 \quad -37x + 13$$
$$\frac{-55x}{-55} = \frac{0}{-55}$$
$$\boxed{x = 0}$$

3. $\overset{10}{\left(0.2(3 - 7x) + 0.9(4x - 3)\right)} = \overset{10}{\left(0.5(2x - 5) + 0.4(3x + 1)\right)}$

$$2(3 - 7x) + 9(4x - 3) = 5(2x - 5) + 4(3x + 1)$$
$$6 - 14x + 36x - 27 = 10x - 25 + 12x + 4$$
$$22x - 21 = 22x - 21$$
$$-22x + 21 \quad -22x + 21$$
$$0 = 0 \quad \checkmark$$
$$\boxed{\text{All real numbers}}$$

4. $8(5x + 2) - 3(7 - 10x) = 5(15x - 4) + 6(5x + 6)$

$40x + 16 - 21 + 30x = 75x - 20 + 30x + 36$

$70x - 5 = 105x + 16$
$-105x + 5 \quad -105x \quad +5$

$\dfrac{-35x}{-35} = \dfrac{21}{-35}$

$\boxed{x = -\frac{3}{5}}$

5. $\overset{12}{\overbrace{}}\left(\frac{5}{12}(7x + 2) + \frac{1}{2}(4 - x)\right) = \overset{12}{\overbrace{}}\left(\frac{1}{4}(9x + 2) - \frac{2}{3}(4x + 5)\right)$

$5(7x+2) + 6(4-x) = 3(9x+2) - 8(4x+5)$

$35x + 10 + 24 - 6x = 27x + 6 - 32x - 40$

$29x + 34 = -5x - 34$
$+5x \quad -34 \quad +5x \quad -34$

$\dfrac{34x}{34} = \dfrac{-68}{34}$

$\boxed{x = -2}$

6. $\overset{45}{\overbrace{}}\left(\frac{2}{5}(5x - 1) - \frac{4}{3}(5 - 3x)\right) = \overset{45}{\overbrace{}}\left(\frac{1}{9}(18x + 5) + \frac{4}{15}(15x - 2)\right)$

$18(5x-1) - 60(5-3x) = 5(18x+5) + 12(15x-2)$

$90x - 18 - 300 + 180x = 90x + 25 + 180x - 24$

$270x - 318 = 270x + 1$
$-270x \qquad -270x$

$-318 = 1 \quad \times$

$\boxed{\text{No solution}}$

Solve.

1. $8(3x - 4) - (9x + 5) = 5(1 - 2x) + 58$

$x = 4$

2. $\dfrac{7x + 1}{9} - \dfrac{4 - 5x}{27} = \dfrac{x - 2}{3} - \dfrac{14x - 51}{81}$

$x = 0$

3. $0.6(2 - 9x) + 1.1(5x - 2) = 0.5(3x - 8) + 0.2(15 - 7x)$

All real numbers

4. $4(25x + 3) - 5(10x - 1) = 7(1 - 15x) + 13(5x - 2)$

$x = -\frac{2}{5}$

5. $\frac{1}{6}(5x - 1) - x = \frac{3}{4}(7x + 3) + \frac{4}{3}(2 - x) - 1$

$x = -1$

6. $\frac{2}{45}(45x + 4) - \frac{1}{9}(7 - 9x) = \frac{1}{3}(1 - 3x) + \frac{4}{5}(5x + 2)$

No solution

Solve. Write each solution set in inequality and graphical notations.

1. $6(2x - 1) + 42x > 5(8x - 3) + 2(4x + 3)$

$12x - 6 + 42x > 40x - 15 + 8x + 6$

$54x - 6 > 48x - 9$
$-48x + 6 \quad\quad -48x + 6$

$\dfrac{6x}{6} > \dfrac{-3}{6}$

$\boxed{x > -\tfrac{1}{2}}$
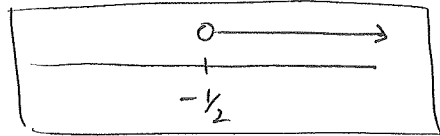
2. $24\left(\dfrac{5x + 1}{8} - \dfrac{11x - 9}{24}\right) \geq 24\left(\dfrac{3x - 1}{4} - \dfrac{15 - x}{12}\right)$

$3(5x+1) - 1(11x - 9) \geq 6(3x - 1) - 2(15 - x)$

$15x + 3 - 11x + 9 \geq 18x - 6 - 30 + 2x$

$4x + 12 \geq 20x - 36$
$-20x - 12 \quad\quad -20x \quad -12$

$\dfrac{-16x}{-16} \leq \dfrac{-48}{-16}$

$\boxed{x \leq 3}$
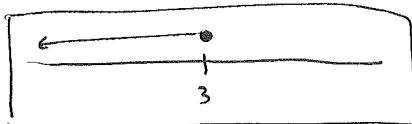
3. $100\left(0.09 - 0.1(15x - 4)\right) \leq 100\left(0.03(7 - 5x) - x\right)$

$9 - 10(15x - 4) \leq 3(7 - 5x) - 100x$

$9 - 150x + 40 \leq 21 - 15x - 100x$

$-150x + 49 \leq -115x + 21$
$+115x - 49 \quad\quad +115x \quad -49$

$\dfrac{-35x}{-35} \geq \dfrac{-28}{-35}$

$\boxed{x \geq \tfrac{4}{5}}$
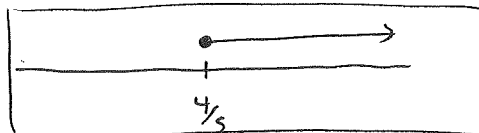
4. $6(3x-2)-5(2x+7)<4(9-4x)-3(1-8x)$

$18x-12-10x-35 < 36-16x-3+24x$

$8x-47 < 8x+33$
$-8x \qquad -8x$

$\qquad -47 < 33 \checkmark$

$\boxed{\text{All real numbers}}$

5. $\left(\overset{49}{\overbrace{\frac{3}{10}(3x+10)}}-3\right) > \left(\overset{49}{\overbrace{\frac{3}{20}(x-10)}}+\frac{1}{8}(5x+2)\right)$

$12(3x+10)-120 > 6(x-10)+5(5x+2)$

$36x+120-120 > 6x-60+25x+10$

$36x > 31x-50$
$-31x \quad -31x$

$\frac{5x}{5} > \frac{-50}{5}$

$\boxed{x > -10}$
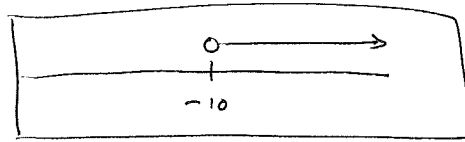
6. $\left(\overset{30}{\overbrace{\frac{4}{15}(30x+11)}}+3x\right) \le \left(\overset{30}{\overbrace{6x}}-\frac{5}{6}(1-6x)\right)$

$8(30x+11)+90x \le 180x-25(1-6x)$

$240x+88+90x \le 180x-25+150x$

$330x+88 \le 330x-25$
$-330x \qquad -330x$

$\qquad 88 \le -25 \quad \times$

$\boxed{\text{No solution}}$

Solve. Write each solution set in inequality and graphical notations.

1. $9(8x + 3) + 48x > 5(16x + 3) - 2(4x + 3)$

$x > -\frac{3}{8}$
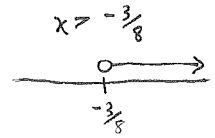
2. $\dfrac{8x + 5}{45} - \dfrac{19x - 5}{90} \geq \dfrac{7x - 17}{18} - \dfrac{35 - x}{30}$

$x \leq 5$
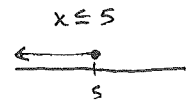
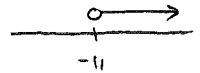3. $0.1(12x - 5) + 0.23 \leq x - 0.09(7 - 4x)$

$x \geq \frac{9}{4}$
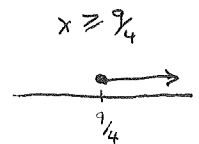
4. $8(13 - 2x) - 5(8 - 5x) < 7(3x + 8) - 6(2x + 3)$

No solution

5. $\frac{1}{36}(5x + 19) + \frac{3}{8}(x - 5) > \frac{5}{12}(x - 1) - 2$

$x > -11$

-11

6. $\frac{1}{2}x - \frac{11}{12}(3 - 2x) \leq \frac{1}{4}(8x - 1) + \frac{1}{3}x$

All real numbers

Solve for the indicated variable.

1. $s = vt - 16t^2$ for v

$$+16t^2 \qquad +16t^2$$

$$\frac{s+16t^2}{t} = \frac{v\cancel{t}}{\cancel{t}}$$

$$\boxed{\frac{s+16t^2}{t} = v}$$

2. $\left(\dfrac{2m+3n}{4}\right)^{20} = \left(\dfrac{6m-5n}{20}\right)^{20}$ for m

$$5(2m+3n) = 1(6m-5n)$$

$$10m + 15n = 6m - 5n$$
$$-6m - 15n \quad -6m \quad -15n$$

$$\frac{\cancel{4}m}{\cancel{4}} = \frac{-20n}{4}$$

$$\boxed{m = -5n}$$

3. $2A = {}^2\!\left(B + \tfrac{1}{2}Ps\right)$ for s

$$2A = 2B + Ps$$
$$-2B \quad -2B$$

$$\frac{2A-2B}{P} = \frac{\cancel{P}s}{\cancel{P}}$$

$$\boxed{\frac{2A-2B}{P} = s}$$

4. $x = \left(\frac{4}{3}y - 1\right)$ for y

$3x = 4y - 3$
$+3 \qquad\quad +3$

$\dfrac{3x + 3}{4} = \dfrac{4y}{4}$

$\boxed{\dfrac{3x + 3}{4} = y}$

5. $4\left(3x^2 - 5y^2\right) - 7\left(6x^2 + y\right) = 2\left(9y - 10y^2\right) - 5\left(7x^2 - y\right)$ for y

$12x^2 - 20y^2 - 42x^2 - 7y = 18y - 20y^2 - 35x^2 + 5y$

$-30x^2 - 20y^2 - 7y = -36x^2 - 20y^2 + 23y$
$+35x^2 + 20y^2 + 7y \qquad +35x^2 + 20y^2 + 7y$

$\dfrac{5x^2}{30} = \dfrac{30y}{30}$

$\boxed{\dfrac{1}{6}x^2 = y}$

6. $m = \left(\dfrac{3x + 8y}{5}\right)$ for x

$5m = 1(3x + 8y)$

$5m = 3x + 8y$
$-8y \qquad\quad -8y$

$\dfrac{5m - 8y}{3} = \dfrac{3x}{3}$

$\boxed{\dfrac{5m - 8y}{3} = x}$

Solve for the indicated variable.

1. $c = ax + bx^2$ for b

$$b = \frac{c - ax}{x^2}$$

2. $\dfrac{3m + 2n}{7} = \dfrac{5m - 2n}{49}$ for m

$$m = -n$$

3. $S = A^2 - \frac{4}{3}PB$ for P

$$P = \frac{3S - 3A^2}{-4B}$$

4. $x = \frac{2}{5}y - 3$ for y

$$y = \frac{5x + 15}{2}$$

5. $9\left(u^2 - 2v^2\right) - 4\left(8u^2 + 5v\right) = 6\left(5v - 3v^2\right) - 15\left(2u^2 + v\right)$ for v

$$v = \frac{1}{5}u^2$$

6. $m = \dfrac{8x + 9y}{10}$ for x

$$x = \frac{10m - 9y}{8}$$

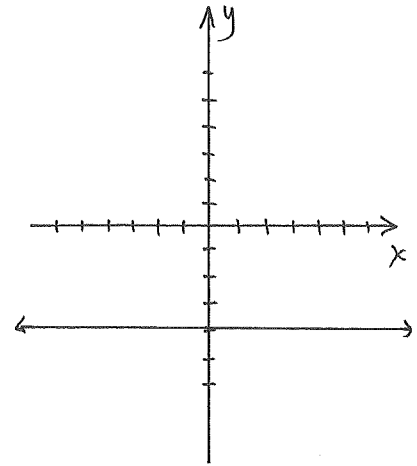# CHAPTER II

LINES AND
LINEAR SYSTEMS

Graph.

1. $5 - 7y = 33$

$$\frac{-7y}{-7} = \frac{28}{-7}$$
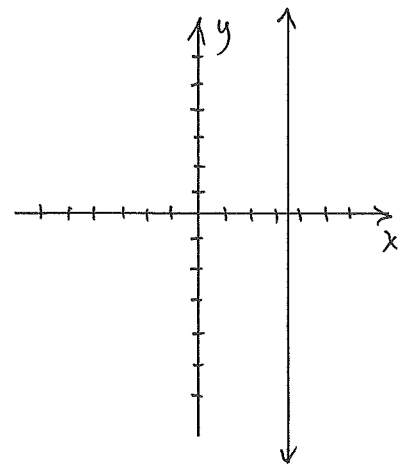
$$\boxed{y = -4}$$

2. $7\left(\frac{2}{7}x + 8\right) = 7(9)$

$2x + 56 = 63$

$$\frac{2x}{2} = \frac{7}{2}$$

$$\boxed{x = \frac{7}{2}}$$

$3\frac{1}{2}$

Find intercepts and graph.

3. $-20x + 8y = -40$

x-int: $-20x + 8(0) = -40$
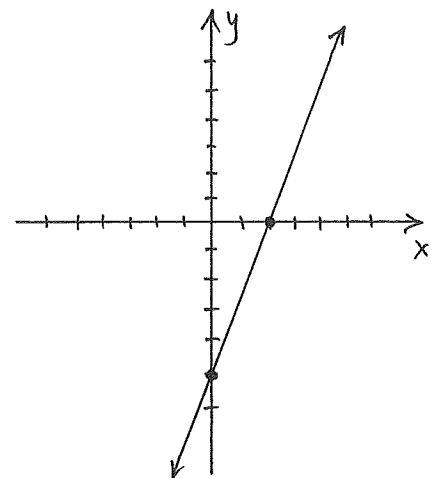
$\boxed{(2, 0)}$ $\frac{-20x}{-20} = \frac{-40}{-20}$

$x = 2$

y-int: $-20(0) + 8y = -40$

$\boxed{(0, -5)}$ $\frac{8y}{8} = \frac{-40}{8}$

$y = -5$

4. $\overset{27}{\left(-\frac{1}{9}x - \frac{4}{27}y\right)} = \overset{27}{\left(-\frac{1}{3}\right)}$

$-3x - 4y = -9$

x-int: $-3x - 4(0) = -9$ y-int: $-3(0) - 4y = -9$

$\boxed{(3, 0)}$ $\dfrac{-3x}{-3} = \dfrac{-9}{-3}$ $\boxed{\left(0, \frac{9}{4}\right)}$ $\dfrac{-4y}{-4} = \dfrac{-9}{-4}$

$x = 3$ $2\frac{1}{4}$ $y = \frac{9}{4}$
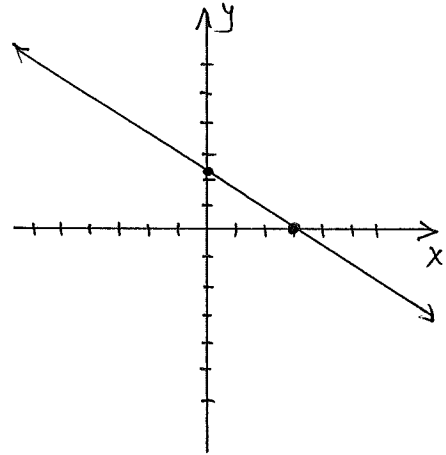
5. $4x - 5y = -10$

x-int: $4x - 5(0) = -10$ y-int: $4(0) - 5y = -10$
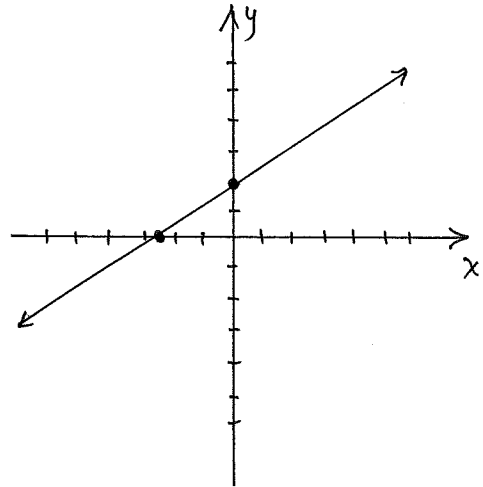
$-2\frac{1}{2}$ $\boxed{\left(-\frac{5}{2}, 0\right)}$ $\dfrac{4x}{4} = \dfrac{-10}{4}$ $\boxed{(0, 2)}$ $\dfrac{-5y}{-5} = \dfrac{-10}{-5}$

$x = -\frac{5}{2}$ $y = 2$

6. $\overset{6}{\left(\frac{10}{3}x + \frac{5}{6}y\right)} = \overset{6}{\left(-\frac{5}{2}\right)}$

$20x + 5y = -15$

x-int: $20x + 5(0) = -15$ y-int: $20(0) + 5y = -15$
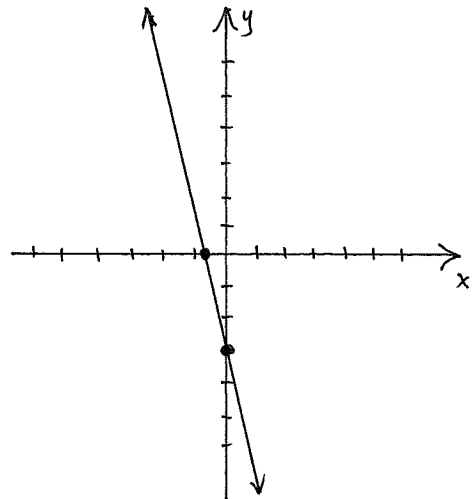
$\boxed{\left(-\frac{3}{4}, 0\right)}$ $\dfrac{20x}{20} = \dfrac{-15}{20}$ $\boxed{(0, -3)}$ $\dfrac{5y}{5} = \dfrac{-15}{5}$

$x = -\frac{3}{4}$ $y = -3$

Graph.

1. $3 - 8y = 19$

2. $\frac{2}{5}x + 10 = 11$

Find intercepts and graph.

3. $-10x + 6y = -30$
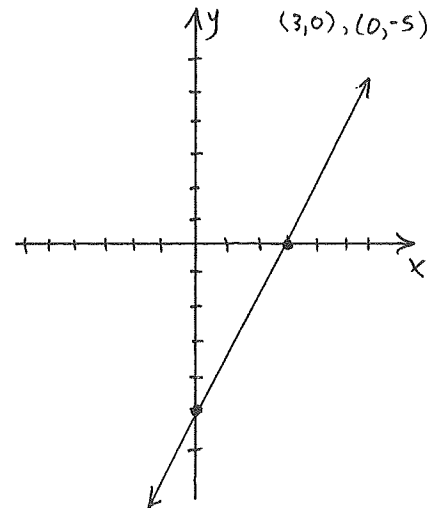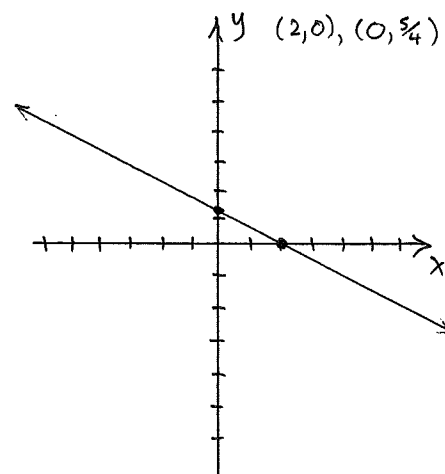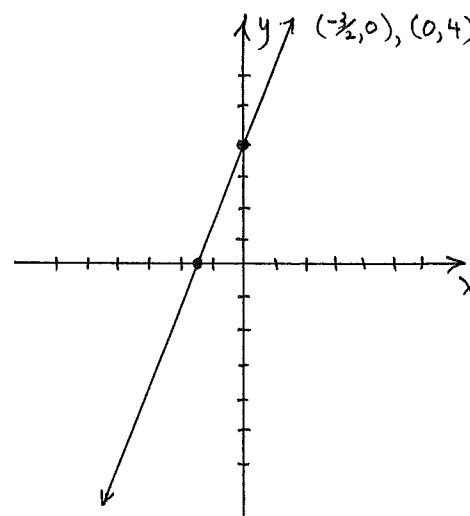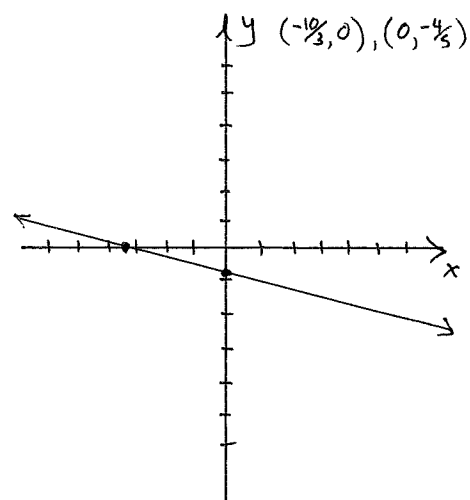
(3,0), (0,-5)

4. $-\frac{5}{32}x - \frac{1}{4}y = -\frac{5}{16}$

$(2,0), (0,\frac{5}{4})$

5. $8x - 3y = -12$

$(-\frac{3}{2},0), (0,4)$

6. $\frac{9}{25}x + \frac{3}{2}y = -\frac{6}{5}$

$(-\frac{10}{3},0), (0,-\frac{4}{5})$

Find the slope of the line passing though the given points. Note whether it rises, falls, is horizontal, or is vertical.

1. $(-27, -28), (-19, 12)$

$$m = \frac{12-(-28)}{-19-(-27)} = \frac{12+28}{-19+27} = \frac{40}{8} = \boxed{5} \quad \boxed{\text{Rises}}$$

2. $\left(-\frac{9}{2}, 15\right), \left(-\frac{9}{2}, 20\right)$

$$m = \frac{20-15}{-\frac{9}{2}-\left(-\frac{9}{2}\right)} = \frac{20-15}{-\frac{9}{2}+\frac{9}{2}} = \frac{5}{0} : \boxed{\text{Undefined}} \quad \boxed{\text{Vertical}}$$

3. $(7, -4), \left(\frac{1}{3}, -\frac{3}{2}\right)$

$$m = \frac{-\frac{3}{2}-(-4)}{\frac{1}{3}-7} = \frac{\left(-\frac{3}{2}+4\right)}{\left(\frac{1}{3}-7\right)} \cdot \frac{6}{6} = \frac{-9+24}{2-42} = \frac{15}{-40} = \boxed{-\frac{3}{8}}$$

Determine whether the given lines L_1 and L_2 are parallel, perpendicular, or neither.

4. L_1 passes through (3, 26) and (13, −14).

   L_2 passes through (51, 42) and (−49, 17).

$$m_1 = \frac{-14-26}{13-3} = \frac{-40}{10} = \boxed{-4}$$

$$m_2 = \frac{17-42}{-49-51} = \frac{-25}{-100} = \boxed{\tfrac{1}{4}}$$

$$\boxed{\text{Perpendicular}}$$

5. L_1 passes through (8, 15) and (−4, 111).

   L_2 has slope $\boxed{-8.}$

$$m_1 = \frac{111-15}{-4-8} = \frac{96}{-12} = \boxed{-8}$$

$$\boxed{\text{Parallel}}$$

6. L_1 passes through (−52, −38) and (−22, −13).

   L_2 passes through (54, 33) and (74, 57).

$$m_1 = \frac{-13-(-38)}{-22-(-52)} = \frac{-13+38}{-22+52} = \frac{25}{30} = \boxed{\tfrac{5}{6}}$$

$$m_2 = \frac{57-33}{74-54} = \frac{24}{20} = \boxed{\tfrac{6}{5}}$$

$$\boxed{\text{Neither}}$$

Find the slope of the line passing though the given points. Note whether it rises, falls, is horizontal, or is vertical.

1. $(-26, -45)$, $(-14, 15)$

5, rises

2. $\left(-\frac{7}{4}, 18\right)$, $\left(-\frac{7}{4}, 39\right)$

Undefined, vertical

3. $(4, -5)$, $\left(\frac{1}{2}, -\frac{1}{3}\right)$

$-\frac{4}{3}$, falls

Determine whether the given lines L_1 and L_2 are parallel, perpendicular, or neither.

4. L_1 passes through $(12, 21)$ and $(37, -29)$.
   L_2 passes through $(46, 59)$ and $(-16, 28)$.

   Perpendicular

5. L_1 passes through $(79, 24)$ and $(68, 101)$.
   L_2 has slope -7.

   Parallel

6. L_1 passes through $(-112, -87)$ and $(-92, -52)$.
   L_2 passes through $(-18, 43)$ and $(-74, 11)$.

   Neither

1. Find and sketch a graph of the slope-intercept form of the line passing through $(-52, -13)$ and $(-12, -3)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \tfrac{1}{4}(x - (-12))$$

$$y + 3 = \tfrac{1}{4}(x + 12)$$

$$y + 3 = \tfrac{1}{4}x + 3$$

$$\boxed{y = \tfrac{1}{4}x}$$

$$m = \frac{-3 - (-13)}{-12 - (-52)} = \frac{-3 + 13}{-12 + 52}$$

$$= \frac{10}{40} = \frac{1}{4}$$

2. Find and sketch a graph of the slope-intercept form of the line passing through $\left(\tfrac{11}{9}, -5\right)$ and $\left(-6, \tfrac{50}{3}\right)$.

$$y - y_1 = m(x - x_1)$$

$$y - \tfrac{50}{3} = -3(x - (-6))$$

$$y - \tfrac{50}{3} = -3(x + 6)$$

$$y - \tfrac{50}{3} = -3x - 18$$

$$y = -3x - 18 + \tfrac{50}{3}$$

$$y = -3x - \tfrac{54}{3} + \tfrac{50}{3}$$

$$\boxed{y = -3x - \tfrac{4}{3}}$$

$$-1\tfrac{1}{3}$$

$$m = \frac{\tfrac{50}{3} - (-5)}{-6 - \tfrac{1}{9}} = \frac{\left(\tfrac{50}{3} + 5\right)}{\left(-6 - \tfrac{1}{9}\right)} \cdot \frac{9}{9}$$

$$= \frac{150 + 45}{-54 - 11} = \frac{195}{-65} = -3$$
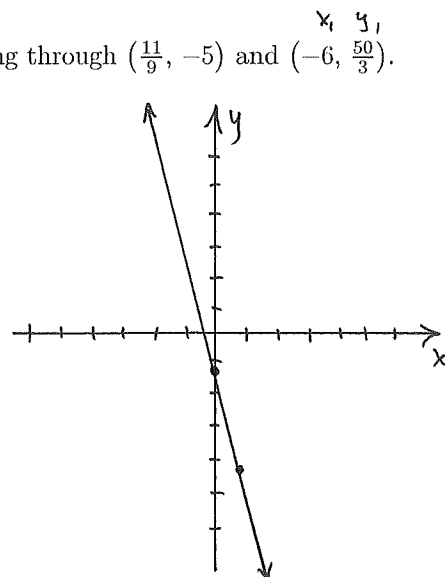
3. Find and sketch a graph of the slope-intercept form of the line passing through $(24, 33)$ and $(-15, -19)$.

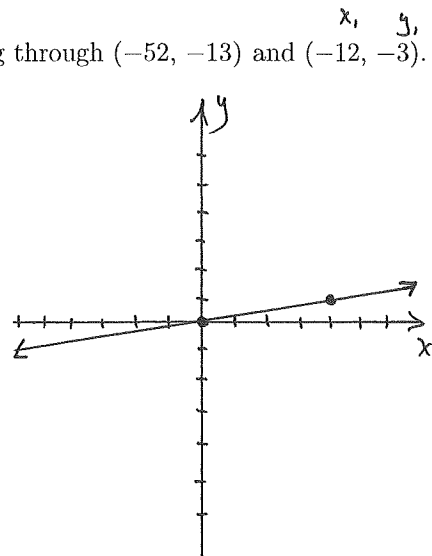$$y - y_1 = m(x - x_1)$$

$$y - 33 = \tfrac{4}{3}(x - 24)$$

$$y - 33 = \tfrac{4}{3}x - 32$$

$$\boxed{y = \tfrac{4}{3}x + 1}$$

$$m = \frac{-19 - 33}{-15 - 24} = \frac{-52}{-39} = \frac{4}{3}$$

4. Find and sketch a graph of the slope-intercept form of the line passing through $\left(-\frac{3}{5}, 3\right)$ and $\left(-\frac{1}{10}, \frac{1}{2}\right)$.

$x_1 \ y_1$

$y - y_1 = m(x - x_1)$

$y - 3 = -5\left(x - \left(-\frac{3}{5}\right)\right)$

$y - 3 = -5\left(x + \frac{3}{5}\right)$

$y - 3 = -5x - 3$

$\boxed{y = -5x}$

$m = \dfrac{\frac{1}{2} - 3}{-\frac{1}{10} - \left(-\frac{3}{5}\right)} = \dfrac{\left(\frac{1}{2} - 3\right)}{\left(-\frac{1}{10} + \frac{3}{5}\right)} \cdot \dfrac{10}{10}$

$= \dfrac{5 - 30}{-1 + 6} = \dfrac{-25}{5} = -5$
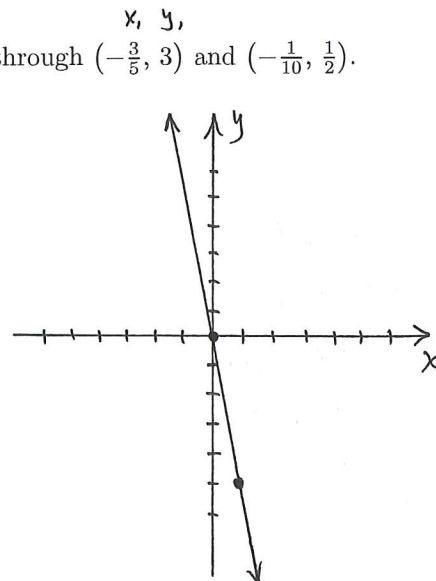
5. Find and sketch a graph of the slope-intercept form of the line passing through $\left(-\frac{7}{4}, -6\right)$ and $\left(\frac{3}{4}, -1\right)$.

$x_1 \ y_1$

$y - y_1 = m(x - x_1)$

$y - (-1) = 2\left(x - \frac{3}{4}\right)$

$y + 1 = 2\left(x - \frac{3}{4}\right)$

$y + 1 = 2x - \frac{3}{2}$

$y = 2x - \frac{3}{2} - 1$
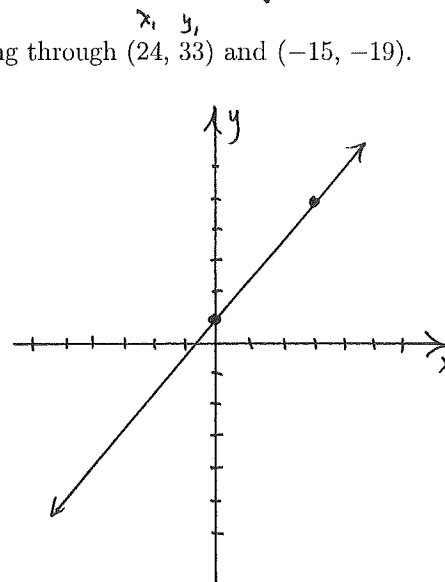
$y = 2x - \frac{3}{2} - \frac{2}{2}$

$\boxed{y = 2x - \frac{5}{2}}$

$-2\frac{1}{2}$

$m = \dfrac{-1 - (-6)}{\frac{3}{4} - \left(-\frac{7}{4}\right)} = \dfrac{(-1 + 6)}{\left(\frac{3}{4} + \frac{7}{4}\right)} \cdot \dfrac{4}{4}$

$= \dfrac{-4 + 24}{3 + 7} = \dfrac{20}{10} = 2$
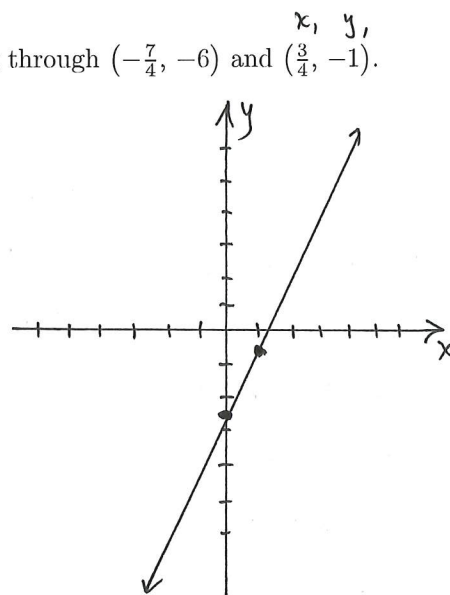
6. Find and sketch a graph of the slope-intercept form of the line passing through $(10, 0)$ and $(-35, 18)$.

$x_1 \ y_1$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{2}{5}(x - 10)$

$y - 0 = -\frac{2}{5}x + 4$

$\boxed{y = -\frac{2}{5}x + 4}$

$m = \dfrac{18 - 0}{-35 - 10} = \dfrac{18}{-45} = -\frac{2}{5}$
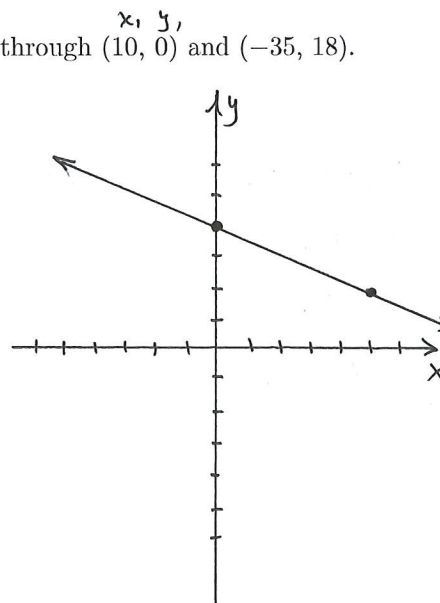
1. Find and sketch a graph of the slope-intercept form of the line passing through $(-65, -13)$ and $(-100, -20)$.

$$y = \tfrac{1}{5}x$$
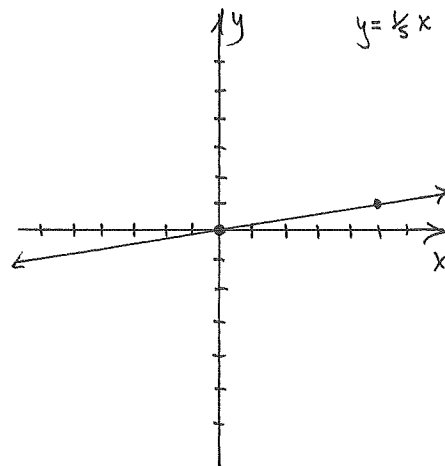
2. Find and sketch a graph of the slope-intercept form of the line passing through $\left(\tfrac{5}{8}, -2\right)$ and $\left(-10, \tfrac{77}{4}\right)$.

$$y = -2x - \tfrac{3}{4}$$
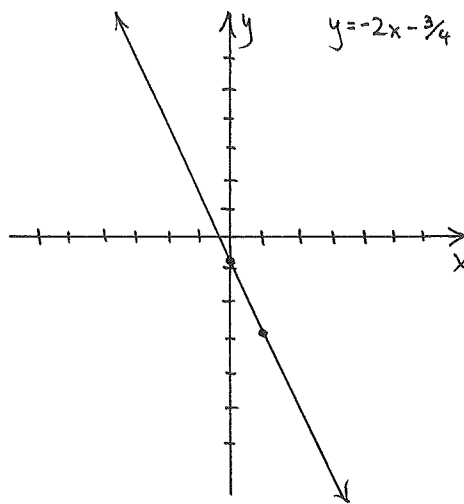
3. Find and sketch a graph of the slope-intercept form of the line passing through $(39, 27)$ and $(-21, -13)$.

$$y = \tfrac{2}{3}x + 1$$
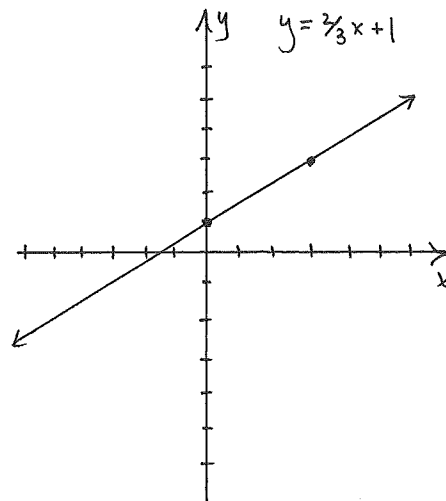
4. Find and sketch a graph of the slope-intercept form of the line passing through $\left(-\frac{7}{4}, 7\right)$ and $\left(-\frac{9}{8}, \frac{9}{2}\right)$.

$y = -4x$
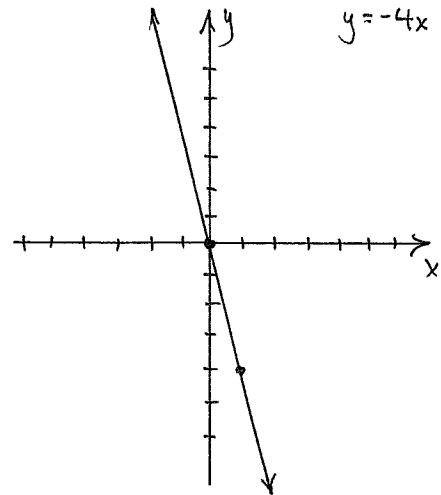
5. Find and sketch a graph of the slope-intercept form of the line passing through $\left(-\frac{5}{6}, -7\right)$ and $\left(\frac{7}{6}, -1\right)$.

$y = 3x - \frac{19}{3}$
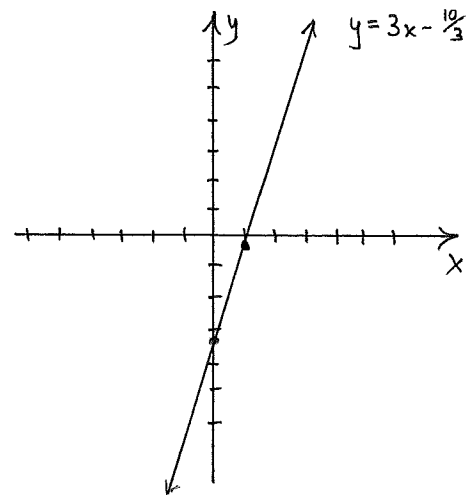
6. Find and sketch a graph of the slope-intercept form of the line passing through $(40, -47)$ and $(-56, 73)$.
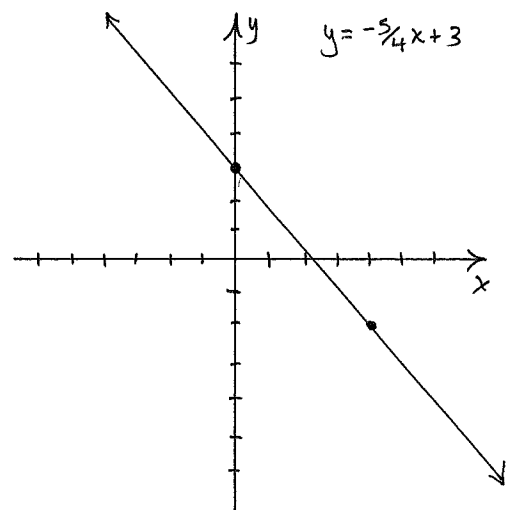
$y = -\frac{5}{4}x + 3$

1. Find and sketch a graph of the slope-intercept form of the line passing through $(-24, 41)$ and parallel a line with slope $-\frac{3}{2}$.   Parallel $m = -\frac{3}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - 41 = -\tfrac{3}{2}(x - (-24))$$
$$y - 41 = -\tfrac{3}{2}(x + 24)$$
$$y - 41 = -\tfrac{3}{2}x - 36$$
$$\boxed{y = -\tfrac{3}{2}x + 5}$$

2. Find and sketch a graph of the slope-intercept form of the line passing through $(19, 73)$ and perpendicular to $x + 4y = 20$.

$$y - y_1 = m(x - x_1)$$
$$y - 73 = 4(x - 19)$$
$$y - 73 = 4x - 76$$
$$\boxed{y = 4x - 3}$$

$$4y = \frac{-x + 20}{4}$$
$$y = -\tfrac{1}{4}x + 5$$

Perpendicular $m = 4$
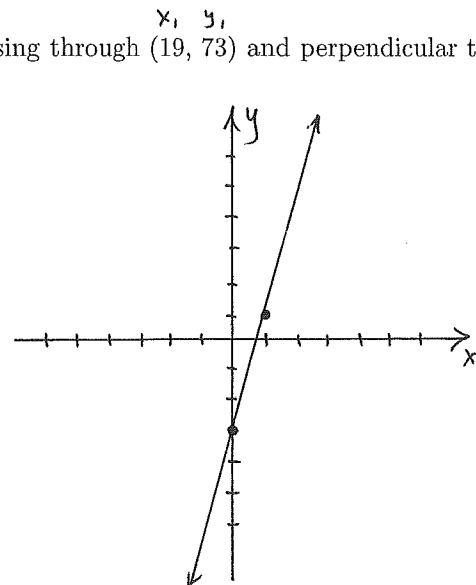
3. Find and sketch a graph of the slope-intercept form of the line passing through $\left(\frac{3}{4}, -\frac{5}{2}\right)$ and parallel to $y = -1$.

Parallel $m = 0$

$$y - y_1 = m(x - x_1)$$
$$y - (-\tfrac{5}{2}) = 0(x - \tfrac{3}{4})$$
$$y + \tfrac{5}{2} = 0$$
$$\boxed{y = -\tfrac{5}{2}}$$
$$-2\tfrac{1}{2}$$

4. Find and sketch a graph of the slope-intercept form of the line passing through $\left(-\frac{45}{4}, 8\right)$ and perpendicular to the line passing through $(-47, -15)$ and $(-3, 51)$.

$x_1 \ y_1$

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{2}{3}\left(x - \left(\frac{-45}{4}\right)\right)$

$y - 8 = -\frac{2}{3}\left(x + \frac{45}{4}\right)$

$y - 8 = -\frac{2}{3}x - \frac{15}{2}$

$y = -\frac{2}{3}x - \frac{15}{2} + 8$

$y = -\frac{2}{3}x - \frac{15}{2} + \frac{16}{2}$

$\boxed{y = -\frac{2}{3}x + \frac{1}{2}}$

$m = \frac{51 - (-15)}{-3 - (-47)} = \frac{51 + 15}{-3 + 47}$

$= \frac{66}{44} = \frac{3}{2}$

Perpendicular $m = -\frac{2}{3}$
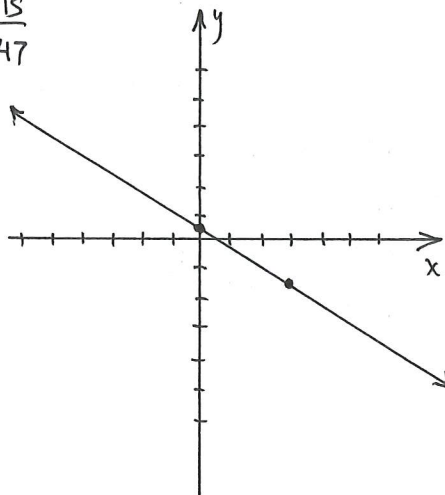
5. Find and sketch a graph of the slope-intercept form of the line passing through $(14, 42)$ and parallel to the line passing through $(-17, 70)$ and $(8, 145)$.

$x_1 \ y_1$

$y - y_1 = m(x - x_1)$

$y - 42 = 3(x - 14)$

$y - 42 = 3x - 42$

$\boxed{y = 3x}$

$m = \frac{145 - 70}{8 - (-17)} = \frac{145 - 70}{8 + 17}$

$= \frac{75}{25} = 3$
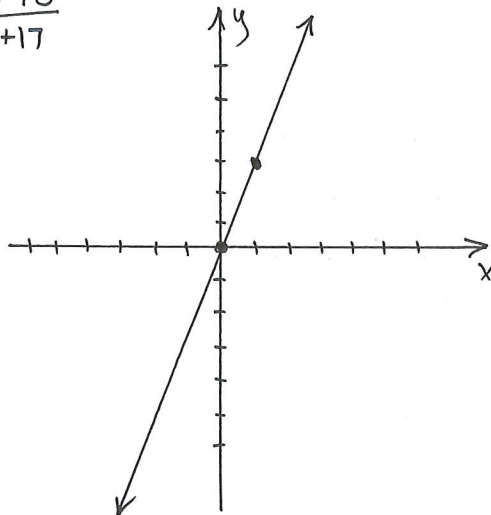
6. Find and sketch a graph of the slope-intercept form of the line passing through $(-2, 4)$ and perpendicular to a vertical line.

$x_1 \ y_1$

Perpendicular $m = 0$

$y - y_1 = m(x - x_1)$

$y - 4 = 0(x - (-2))$

$y - 4 = 0$

$\boxed{y = 4}$
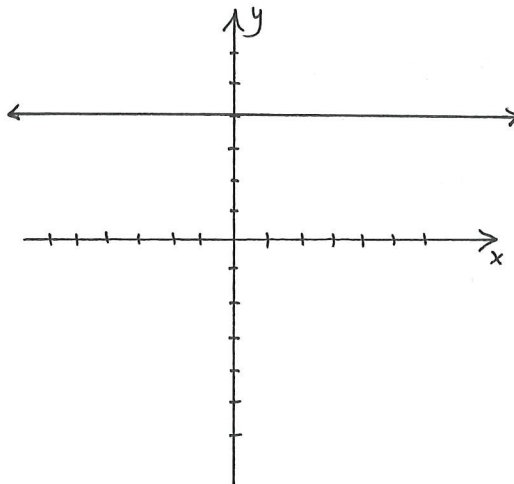
1. Find and sketch a graph of the slope-intercept form of the line passing through $(-48, 69)$ and parallel a line with slope $-\frac{4}{3}$.

$y = -\frac{4}{3}x + 5$
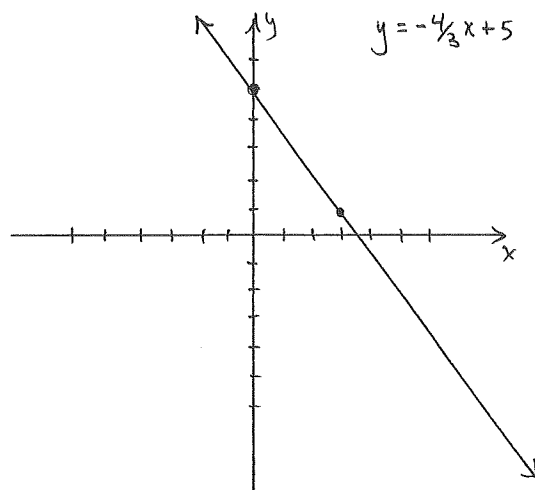
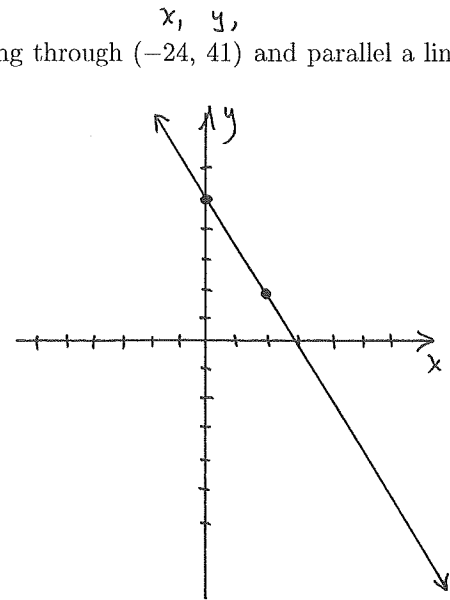2. Find and sketch a graph of the slope-intercept form of the line passing through $(28, 139)$ and perpendicular to $x + 5y = -90$.

$y = 5x - 1$
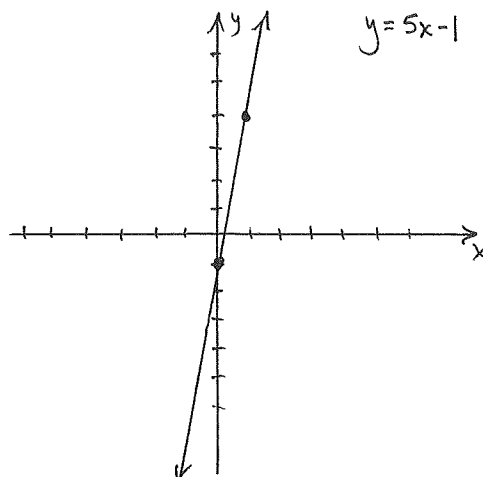
3. Find and sketch a graph of the slope-intercept form of the line passing through $\left(\frac{7}{8}, -\frac{3}{4}\right)$ and parallel to $y = -3$.

$y = -\frac{3}{4}$
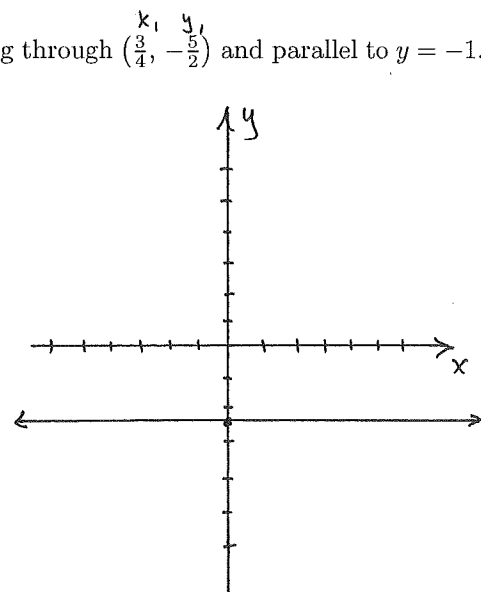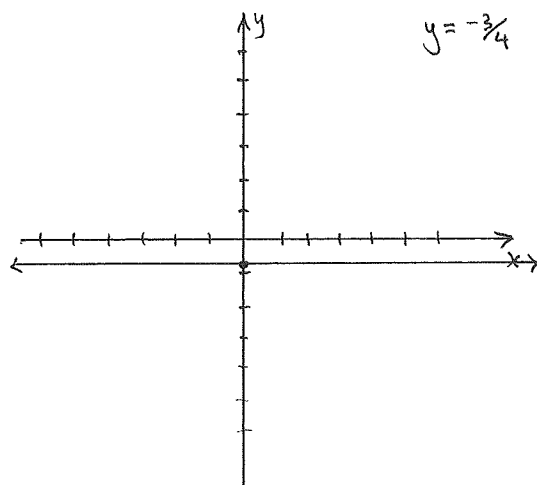
4. Find and sketch a graph of the slope-intercept form of the line passing through $\left(-\frac{3}{10}, 1\right)$ and perpendicular to the line passing through $(-7, 34)$ and $(-17, 30)$.

$y = -\frac{5}{2}x + \frac{1}{4}$
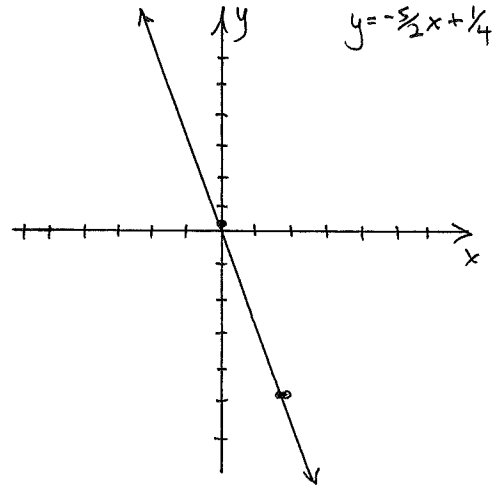
5. Find and sketch a graph of the slope-intercept form of the line passing through $(23, 92)$ and parallel to the line passing through $(-38, 145)$ and $(37, 445)$.

$y = 4x$
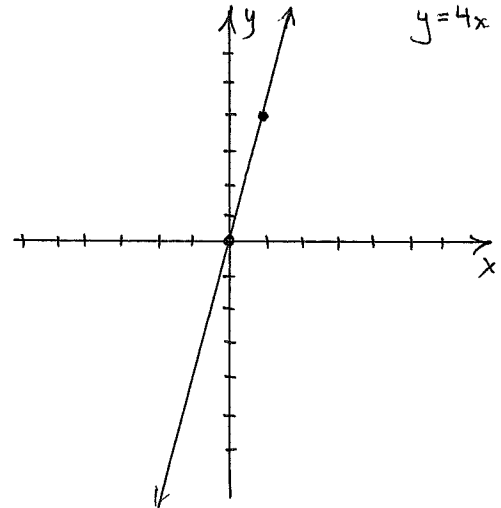
6. Find and sketch a graph of the slope-intercept form of the line passing through $(-8, 3)$ and perpendicular to a vertical line.
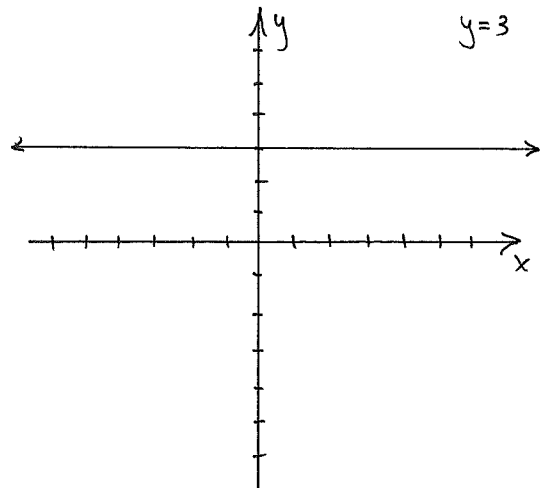
$y = 3$

Solve by elimination.

1. $\begin{cases} 3x + 10y = 82 \\ 6x - 5y = -11 \end{cases}$ $\xrightarrow{\Longleftrightarrow}$ (2)

$$3x + 10y = 82$$
$$12x - 10y = -22$$
$$\overline{}$$
$$\frac{15x}{15} = \frac{60}{15}$$
$$\boxed{x = 4}$$

$$6(4) - 5y = -11$$
$$24 - 5y = -11$$
$$\frac{-5y}{-5} = \frac{-35}{-5}$$
$$\boxed{y = 7}$$

$$\boxed{(4, 7)}$$

2. $\begin{cases} 4x - 9y = 4 \\ -12x - 15y = 16 \end{cases}$ $\overset{(3)}{\Longleftrightarrow}$

$$12x - 27y = 12$$
$$-12x - 15y = 16$$
$$\overline{}$$
$$\frac{-42y}{-42} = \frac{28}{-42}$$
$$\boxed{y = -\frac{2}{3}}$$

$$4x - 9\left(-\frac{2}{3}\right) = 4$$
$$4x + 6 = 4$$
$$\frac{4x}{4} = \frac{-2}{4}$$
$$\boxed{x = -\frac{1}{2}}$$

$$\boxed{\left(-\frac{1}{2}, -\frac{2}{3}\right)}$$

3. $\begin{cases} -\frac{3}{5}x - \frac{2}{3}y = -\frac{1}{2} \\ -\frac{1}{2}x - \frac{5}{9}y = -\frac{7}{12} \end{cases}$ $\overset{(30)}{\underset{(36)}{\Longleftrightarrow}}$

$$-18x - 20y = -15$$
$$-18x - 20y = -21$$

$\overset{(-1)}{\Longleftrightarrow}$

$$18x + 20y = 15$$
$$-18x - 20y = -21$$
$$\overline{}$$
$$0 = -6 \quad \times$$

$$\boxed{\text{No solution}}$$

4. $\begin{cases} \frac{1}{8}x + \frac{1}{3}y = -\frac{13}{24} \\ -11x + 20y = 60 \end{cases}$ $\overset{(24)}{\Longleftrightarrow}$ $\begin{array}{l} 3x + 8y = -13 \\ -11x + 20y = 60 \end{array}$ $\overset{(-5)}{\underset{(2)}{\Longleftrightarrow}}$ $\begin{array}{l} -15x - 40y = 65 \\ -22x + 40y = 120 \end{array}$ $\qquad$ $-11(-5) + 20y = 60$

$$\frac{-37x}{-37} = \frac{185}{-37} \qquad \qquad 55 + 20y = 60$$

$$\boxed{x = -5} \qquad\qquad \frac{20y}{20} = \frac{5}{20}$$

$$\boxed{y = \frac{1}{4}}$$

$$\boxed{(-5, \frac{1}{4})}$$

5. $\begin{cases} 9x + 17y = 3 \\ 12x + 19y = 15 \end{cases}$ $\overset{(4)}{\underset{(-3)}{\Longleftrightarrow}}$ $\begin{array}{l} 36x + 68y = 12 \\ -36x - 57y = -45 \end{array}$ $\qquad$ $9x + 17(-3) = 3$

$$\frac{11y}{11} = \frac{-33}{11} \qquad\qquad 9x - 51 = 3$$

$$\boxed{y = -3} \qquad\qquad \frac{9x}{9} = \frac{54}{9}$$

$$\boxed{x = 6}$$

$$\boxed{(6, -3)}$$

6. $\begin{cases} -\frac{3}{14}x + \frac{1}{2}y = \frac{1}{4} \\ 24x - 56y = -28 \end{cases}$ $\overset{(28)}{\Longleftrightarrow}$ $\begin{array}{l} -6x + 14y = 7 \\ 24x - 56y = -28 \end{array}$ $\overset{(4)}{\Longleftrightarrow}$ $\begin{array}{l} -24x + 56y = 28 \\ 24x - 56y = -28 \end{array}$

$$0 = 0 \checkmark$$

$$\boxed{\text{Infinitely many solutions}}$$

Solve by elimination.

1. $\begin{cases} 4x + 5y = 58 \\ 2x - 15y = -146 \end{cases}$

$(2, 10)$

2. $\begin{cases} 6x - 15y = -10 \\ -18x - 12y = 49 \end{cases}$

$(-\frac{5}{2}, -\frac{1}{3})$

3. $\begin{cases} -\frac{3}{5}x - \frac{4}{25}y = -\frac{4}{15} \\ -\frac{15}{16}x - \frac{1}{4}y = \frac{9}{24} \end{cases}$

No solution

4. $\begin{cases} \frac{7}{50}x + \frac{6}{25}y = -\frac{4}{5} \\ -25x + 20y = 190 \end{cases}$ $(-7, \frac{3}{4})$

5. $\begin{cases} 10x + 11y = -15 \\ 45x + 17y = 95 \end{cases}$ $(4, -5)$

6. $\begin{cases} -\frac{3}{2}x + \frac{5}{3}y = \frac{5}{4} \\ 18x - 20y = -15 \end{cases}$ Infinitely many solutions

CHAPTER III

POLYNOMIAL OPERATIONS
AND FUNCTIONS

Simplify.

1. $\left(7xy^5\right)\left(4x^3y^8\right)$

$= \boxed{28x^4y^{13}}$

2. $\dfrac{-96x^{11}z^{27}}{8x^{18}y^0z^{22}}$

$= \dfrac{-96x^{11}z^{27}}{8x^{18}(1)z^{22}}$

$= \dfrac{-96x^{11}z^{27}}{8x^{18}z^{22}}$

$= \boxed{\dfrac{-12z^5}{x^7}}$

3. $\left(-2x^8y^7\right)\left(-3x^5y^{12}\right)^3$

$= \left(-2x^8y^7\right)\left(-27x^{15}y^{36}\right)$

$= \boxed{54x^{23}y^{43}}$

4. $\dfrac{5x^3y^2}{-(-5x^6y^{10})^2}$

$= \dfrac{5x^3y^2}{-(25x^{12}y^{20})}$

$= \dfrac{5x^3y^2}{-25x^{12}y^{20}}$

$= \boxed{\dfrac{1}{-5x^9y^{18}}}$

5. $\left(\dfrac{32x^{27}y^{18}}{40x^{19}y^{25}}\right)^3$

$= \left(\dfrac{4x^8}{5y^7}\right)^3$

$= \boxed{\dfrac{64x^{24}}{125y^{21}}}$

6. $-\left(6x^{31}y^0z^{18}\right)^2$

$= -\left(6x^{31}(1)z^{18}\right)^2$

$= -\left(6x^{31}z^{18}\right)^2$

$= -\left(36x^{62}z^{36}\right)$

$= \boxed{-36x^{62}z^{36}}$

Simplify.

1. $\left(9xy^7\right)\left(8x^4y^9\right)$

$72x^5y^{16}$

2. $\dfrac{-120x^{14}z^{35}}{15x^{23}y^0z^{18}}$

$\dfrac{-8z^{17}}{x^9}$

3. $\left(-4x^7y^{28}\right)\left(-5x^{14}y^6\right)^3$

$500x^{49}y^{46}$

4. $\dfrac{6x^{16}y^3}{-\left(-6x^{11}y^7\right)^2}$

$\dfrac{1}{-6x^6y^{11}}$

5. $\left(\dfrac{54x^{31}y^{25}}{45x^{16}y^{29}}\right)^3$

$\dfrac{216x^{45}}{125y^{12}}$

6. $-\left(12x^{27}y^0z^{39}\right)^2$

$-144x^{54}y^{78}$

Simplify.

1. $\left(5x^{17}y^{-12}\right)\left(-14x^{-32}y^{-1}\right)$

$$= \frac{-70x^{-15}y^{-13}}{1}$$

$$= \boxed{\frac{-70}{x^{15}y^{13}}}$$

2. $\dfrac{49x^{-11}y^{28}}{42x^{-2}y^{20}}$

$$= \frac{49x^{2}y^{28}}{42x^{11}y^{20}}$$

$$= \boxed{\frac{7y^{8}}{6x^{9}}}$$

3. $\dfrac{\left(8x^{35}z^{-4}\right)\left(2x^{16}y^{0}z^{-9}\right)^{-5}}{1}$

$$= \frac{\left(8x^{35}z^{-4}\right)}{\left(2x^{16}y^{0}z^{-9}\right)^{5}}$$

$$= \frac{8x^{35}z^{-4}}{\left(2x^{16}z^{-9}\right)^{5}}$$

$$= \frac{8x^{35}z^{-4}}{32x^{80}z^{-45}}$$

$$= \frac{8x^{35}z^{45}}{32x^{80}z^{4}} = \boxed{\frac{z^{41}}{4x^{45}}}$$

4. $\dfrac{\left(x^{-7}y^5\right)^{-4}}{x^{25}y^{-19}}$

$= \dfrac{1}{\left(x^{25}y^{-19}\right)\left(x^{-7}y^5\right)^4}$

$= \dfrac{1}{\left(x^{25}y^{-19}\right)\left(x^{-28}y^{20}\right)}$

$= \dfrac{1}{x^{-3}y}$

$= \boxed{\dfrac{x^3}{y}}$

5. $\left(\dfrac{x^{11}y^8}{-9x^{26}y^3}\right)^{-2}$

$= \left(\dfrac{-9x^{26}y^3}{x^{11}y^8}\right)^2$

$= \left(\dfrac{-9x^{15}}{y^5}\right)^2$

$= \boxed{\dfrac{81x^{30}}{y^{10}}}$

6. $\left(\dfrac{84x^2y^{-6}}{-14x^{13}y^{15}}\right)^{-3}$

$= \left(\dfrac{-14x^{13}y^{15}}{84x^2y^{-6}}\right)^3$

$= \left(\dfrac{-14x^{13}y^{15}y^6}{84x^2}\right)^3$

$= \left(\dfrac{-x^{11}y^{21}}{6}\right)^3$

$= \boxed{\dfrac{-x^{33}y^{63}}{216}}$

Simplify.

1. $\left(8x^{21}y^{-9}\right)\left(-15x^{-35}y^{-7}\right)$

$$\frac{-120}{x^{14}y^{16}}$$

2. $\dfrac{72x^{-7}y^{-24}}{63x^{-11}y}$

$$\frac{8x^{4}}{7y^{25}}$$

3. $\left(9x^{59}z^{-5}\right)\left(3x^{18}y^{0}z^{-11}\right)^{-4}$

$$\frac{z^{39}}{9x^{13}}$$

4. $\dfrac{\left(2x^8y^{-6}\right)^{-5}}{x^{-30}y^{16}}$

$\dfrac{y^{14}}{32x^{10}}$

5. $\left(\dfrac{-40x^{25}y^{40}}{45x^{13}y^{49}}\right)^{-2}$

$\dfrac{81y^{18}}{64x^{24}}$

6. $\left(\dfrac{125x^{-6}y^{-13}}{-100x^{19}y^{-11}}\right)^{-3}$

$\dfrac{-64x^{75}y^{6}}{125}$

Simplify.

1. $\left(14x^3 - 17x^2y + 9xy^2 + 11y^3\right) - \left(9x^3 - 3x^2y - 4y^3\right) + \left(3x^3 + 7x^2y - 3xy^2 - 20y^3\right)$

$= 14x^3 - 17x^2y + 9xy^2 + 11y^3 - 9x^3 + 3x^2y + 4y^3 + 3x^3 + 7x^2y - 3xy^2 - 20y^3$

$= \boxed{8x^3 - 7x^2y + 6xy^2 - 5y^3}$

Multiply.

2. $-3x^5y^2\left(9x^2 - 4xy - 8y^2\right)$

$= \boxed{-27x^7y^2 + 12x^6y^3 + 24x^5y^4}$

3. $(8x + 7)(5x - 9)$

$= 40x^2 - 72x + 35x - 63$

$= \boxed{40x^2 - 37x - 63}$

4. $(12x + 5y)(12x - 5y)$

$= 144x^2 = 60xy + 60xy - 25y^2$

$= \boxed{144x^2 - 25y^2}$

5. $(7x - 10)^2$

$= (7x - 10)(7x - 10)$

$= 49x^2 - 70x - 70x + 100$

$= \boxed{49x^2 - 140x + 100}$

6. $(3x - 4y)(5x^2 - 2xy + 8y^2)$

$= 15x^3 - 6x^2y + 24xy^2 - 20x^2y + 8xy^2 - 32y^3$

$= \boxed{15x^3 - 26x^2y + 32xy^2 - 32y^3}$

Simplify.

1. $\left(9x^3 - 15x^2y + 4xy^2 + 7y^3\right) - \left(14x^3 - 2x^2y - 5y^3\right) + \left(4x^3 + 15x^2y - 7xy^2 - 8y^3\right)$ $-x^3 + 2x^2y - 3xy^2 + 4y^3$

Multiply.

2. $-9x^4y^7\left(5x^2 - 8xy + 12y^2\right)$ $-45x^6y^7 + 72x^5y^8 - 108x^4y^9$

3. $(15x - 2)(6x + 5)$ $90x^2 + 63x - 10$

4. $(14x + 13y)(14x - 13y)$

$196x^2 - 169y^2$

5. $(10x - 7)^2$

$100x^2 - 140x + 49$

6. $(7x - 4y)\left(6x^2 - 9xy - 2y^2\right)$

$42x^3 - 87x^2y + 22xy^2 + 8y^3$

Divide.

1. $\dfrac{24x^6y^7 - 15x^2y^6 + 2xy}{6x^3y^5}$

$= \dfrac{24x^6y^7}{6x^3y^5} - \dfrac{15x^2y^6}{6x^3y^5} + \dfrac{2xy}{6x^3y^5}$

$= \boxed{4x^3y^2 - \dfrac{5y}{2x} + \dfrac{1}{3x^2y^4}}$

2. $\left(2x^2 - 5x + 4\right) \div (x - 3)$

$$
\begin{array}{r}
2x + 1 \\
(x-3) \overline{\smash{\big)}\ 2x^2 - 5x + 4} \\
\ominus 2x^2 \oplus 6x \downarrow \\
\hline
x + 4 \\
\ominus x \oplus 3 \\
\hline
7
\end{array}
$$

$\boxed{2x + 1 + \dfrac{7}{x-3}}$

3. $\dfrac{11x^2 - 37x + 4x^3 - 13}{x + 5}$

$$
\begin{array}{r}
4x^2 - 9x + 8 \\
(x+5) \overline{\smash{\big)}\ 4x^3 + 11x^2 - 37x - 13} \\
\ominus 4x^3 \oplus 20x^2 \downarrow \\
\hline
-9x^2 - 37x \\
\oplus 9x^2 \ominus 45x \downarrow \\
\hline
8x - 13 \\
\ominus 8x \oplus 40 \\
\hline
-53
\end{array}
$$

$\boxed{4x^2 - 9x + 8 + \dfrac{-53}{x+5}}$

4. $(7x^3 - 71x^2 + 103) \div (x - 10)$

$$
\begin{array}{r}
7x^2 - x - 10 \\
(x-10) \overline{)7x^3 - 71x^2 + 0x + 103} \\
\ominus 7x^3 \oplus 70x^2 \downarrow \\
-x^2 + 0x \\
\oplus x^2 \ominus 10x \\
\hline
-10x + 103 \\
\oplus 10x \mp 100 \\
\hline
3
\end{array}
$$

$$\boxed{7x^2 - x - 10 + \dfrac{3}{x-10}}$$

5. $\dfrac{118 - 66x^2 + x^4}{x + 8}$

$$
\begin{array}{r}
x^3 - 8x^2 - 2x + 16 \\
(x+8) \overline{)x^4 + 0x^3 - 66x^2 + 0x + 118} \\
\ominus x^4 \oplus 8x^3 \downarrow \\
-8x^3 - 66x^2 \\
\oplus 8x^3 \oplus 64x^2 \downarrow \\
\hline
-2x^2 + 0x \\
\oplus 2x^2 \oplus 16x \downarrow \\
\hline
16x + 118 \\
\ominus 16x \ominus 128 \\
\hline
-10
\end{array}
$$

$$\boxed{x^3 - 8x^2 - 2x + 16 + \dfrac{-10}{x+8}}$$

6. $(x^5 - 1) \div (x - 1)$

$$
\begin{array}{r}
x^4 + x^3 + x^2 + x + 1 \\
(x-1) \overline{)x^5 + 0x^4 + 0x^3 + 0x^2 + 0x - 1} \\
\ominus x^5 \oplus x^4 \downarrow \\
x^4 + 0x^3 \\
\ominus x^4 \oplus x^3 \downarrow \\
x^3 + 0x^2 \\
\ominus x^3 \oplus x^2 \downarrow \\
x^2 + 0x \\
\ominus x^2 \oplus x \downarrow \\
x - 1 \\
\ominus x \oplus 1 \\
\hline
0
\end{array}
$$

$$\boxed{x^4 + x^3 + x^2 + x + 1}$$

Divide.

1. $\dfrac{18x^9y + 45x^3y^6 - 21xy^2}{9x^2y^4}$

$\dfrac{2x^7}{y^3} + 5xy^2 - \dfrac{7}{3xy^2}$

2. $\left(2x^2 - 9x - 10\right) \div (x - 6)$

$2x + 3 + \dfrac{8}{x-6}$

3. $\dfrac{134x^2 - 12x + 15x^3 - 29}{x + 9}$

$15x^2 - x - 3 + \dfrac{-2}{x+9}$

4. $\left(4x^3 - 53x^2 + 723\right) \div (x - 12)$

$4x^2 - 5x - 60 + \dfrac{3}{x-12}$

5. $\dfrac{46 - 27x^2 + x^4}{x + 5}$

$x^3 - 5x^2 - 2x + 10 + \dfrac{-4}{x+5}$

6. $\left(x^5 - 243\right) \div (x - 3)$

$x^4 + 3x^3 + 9x^2 + 27x + 81$

1. Sketch a graph that represents a function. Use the Vertical Line Test to justify your choice.
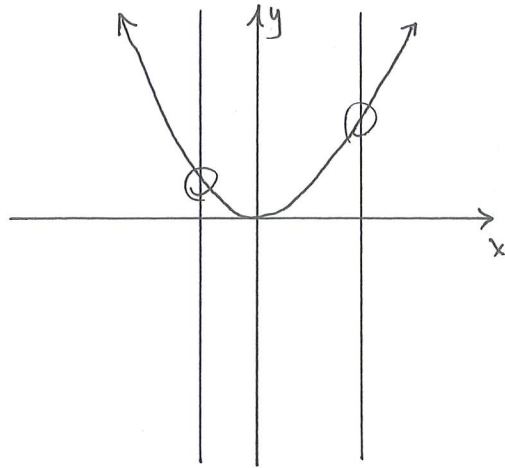
This graph passes VLT, so it is a function.

2. Is a circle a function? Explain why or why not.

No. A circle will fail VLT.

State the domain and range of the given graphed function.

3.

////// : in domain

{ : in range

D: ℝ

R: ℝ

4.
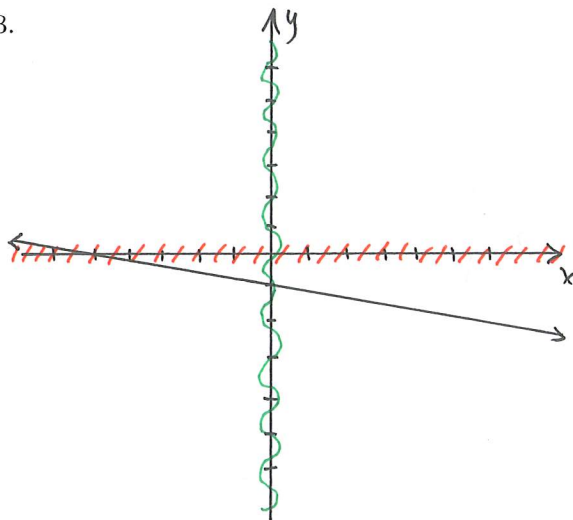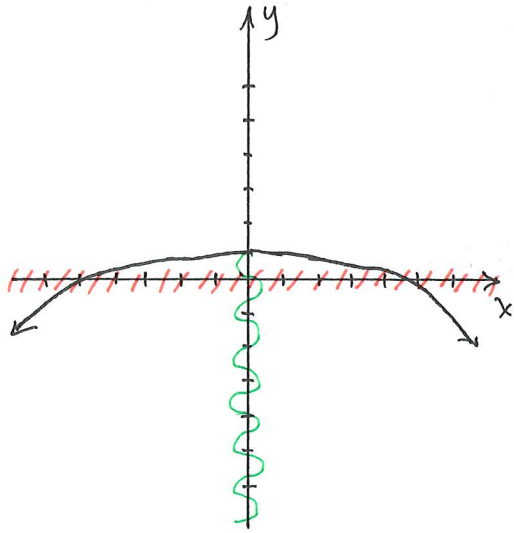
D: ℝ
R: $y \le 1$

//////// : in domain

{ : in range

5.
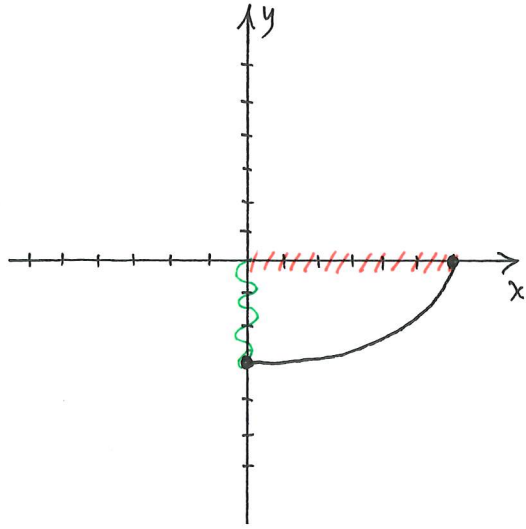
D: $0 \le x \le 6$
R: $-3 \le y \le 0$

6.
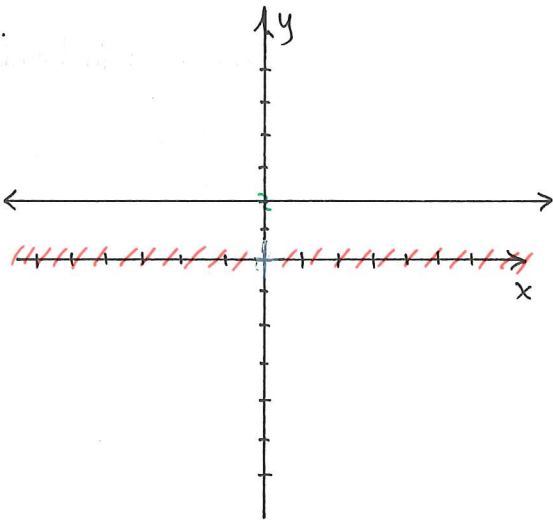
D: ℝ
R: $y = 2$

1. Sketch a graph that does not represent a function. Use the Vertical Line Test to justify your choice.

Graphs will vary.
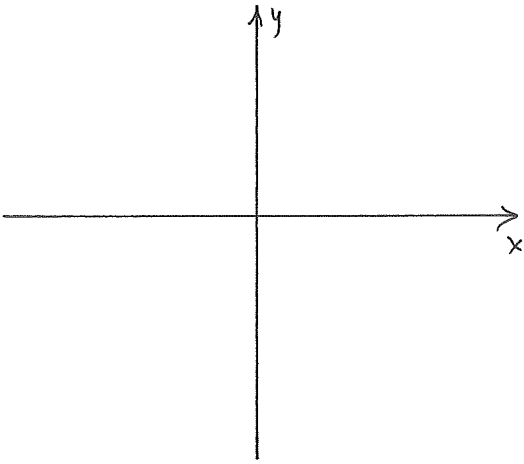
2. Is every parabola a function? Explain why or why not.

No. Not every parabola passes Vertical Line Test.

State the domain and range of the given graphed function.

3.
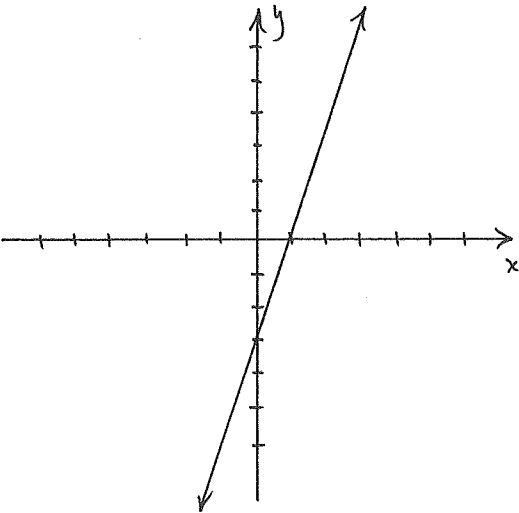
D: $\mathbb{R}$

R: $\mathbb{R}$

4.
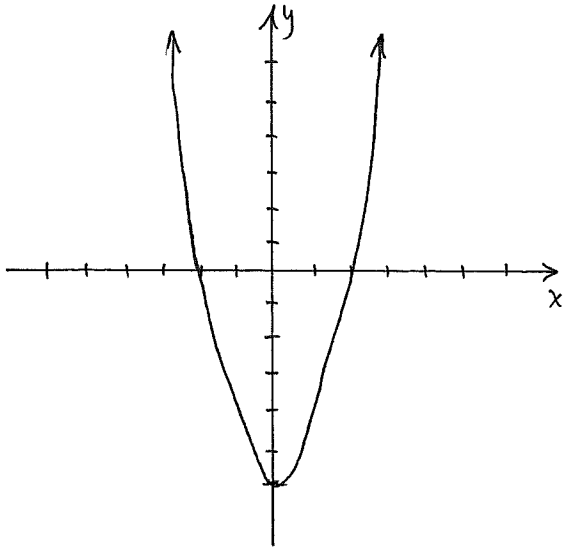
$D: \mathbb{R}$

$R: y \geq -6$

5.
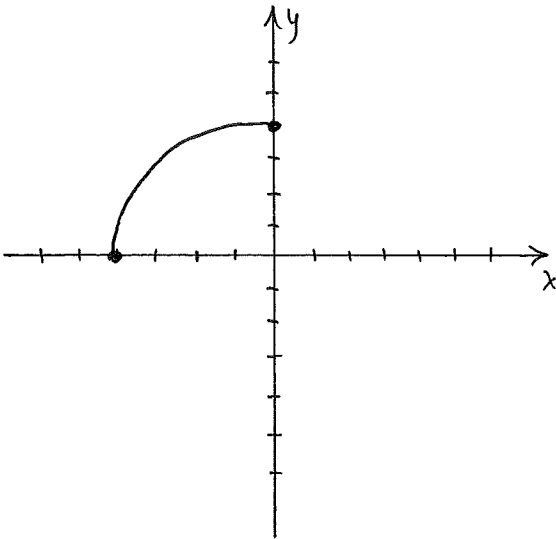
$D: -4 \leq x \leq 0$

$R: 0 \leq y \leq 4$

6.
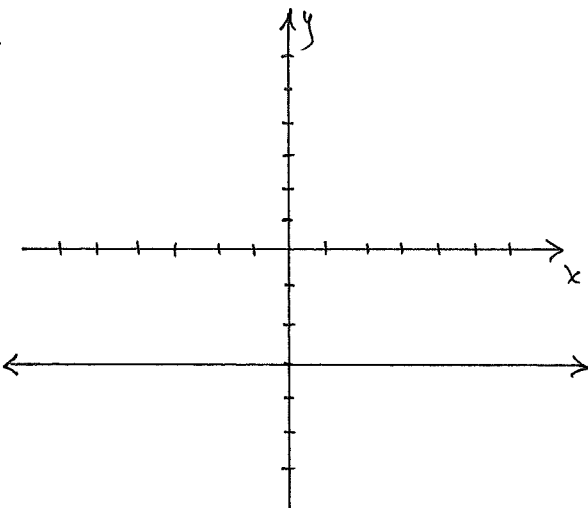
$D: \mathbb{R}$

$R: y = -3$

1. Let $f(x) = 5x - 4$.

   a. State the domain of f. $\boxed{\mathbb{R}}$

   b. Evaluate $f(-7)$.

   $f(-7) = 5(-7) - 4 = -35 - 4 = \boxed{-39}$

2. Let $g(x) = 8x^2 + 7x - 2$.

   a. State the domain of g. $\boxed{\mathbb{R}}$

   b. Evaluate $g(5)$.

   $g(5) = 8(5)^2 + 7(5) - 2 = 8(25) + 7(5) - 2 = 200 + 35 - 2 = \boxed{233}$

3. Let $h(x) = \dfrac{5x + 4}{3x - 2}$.

   $3x - 2 = 0$

   $\dfrac{3x}{3} = \dfrac{2}{3}$

   $\boxed{x \neq \frac{2}{3}}$

   a. State the domain of h.

   b. Evaluate $h(-2)$.

   $h(-2) = \dfrac{5(-2) + 4}{3(-2) - 2} = \dfrac{-10 + 4}{-6 - 2} = \dfrac{-6}{-8} = \boxed{\dfrac{3}{4}}$

4. Let $j(x) = -4 - \frac{8}{3}x$.

   a. State the domain of j. $\boxed{\mathbb{R}}$

   b. Evaluate $j\left(\frac{9}{16}\right)$.

$$j\left(\frac{9}{16}\right) = -4 - \frac{8}{3}\left(\frac{9}{16}\right) = -\frac{4}{1} - \frac{3}{2} = -\frac{8}{2} - \frac{3}{2} = \boxed{-\frac{11}{2}}$$

5. Let $k(x) = -x^2 - 4x + 1$.

   a. State the domain of k. $\boxed{\mathbb{R}}$

   b. Evaluate $k\left(-\frac{4}{3}\right)$.

$$k\left(-\frac{4}{3}\right) = -\left(-\frac{4}{3}\right)^2 - 4\left(-\frac{4}{3}\right) + 1 = -\left(\frac{16}{9}\right) - 4\left(-\frac{4}{3}\right) + 1 = -\frac{16}{9} + \frac{16}{3} + 1 = -\frac{16}{9} + \frac{48}{9} + \frac{9}{9} = \boxed{\frac{41}{9}}$$

6. Let $l(x) = \dfrac{3x - 5}{2x + 5}$.

$$2x + 5 = 0$$
$$\frac{2x}{2} = \frac{-5}{2}$$
$$\boxed{x \neq -\frac{5}{2}}$$

   a. State the domain of l.

   b. Evaluate $l\left(\frac{5}{2}\right)$.

$$l\left(\frac{5}{2}\right) = \frac{3\left(\frac{5}{2}\right) - 5}{2\left(\frac{5}{2}\right) + 5} = \frac{\frac{15}{2} - 5}{5 + 5} = \frac{\left(\frac{15}{2} - 5\right)}{10} \cdot \frac{2}{2} = \frac{15 - 10}{20} = \frac{5}{20} = \boxed{\frac{1}{4}}$$

1. Let $f(x) = 4x - 5$.

   a. State the domain of f. $\mathbb{R}$

   b. Evaluate $f(-9)$. -41

2. Let $g(x) = 7x^2 - 8x + 2$.

   a. State the domain of g. $\mathbb{R}$

   b. Evaluate $g(4)$. 82

3. Let $h(x) = \dfrac{8x + 3}{9x - 1}$.

   a. State the domain of h. $x \neq \frac{1}{9}$

   b. Evaluate $h(-1)$. $\frac{1}{2}$

4. Let $j(x) = -6 - \frac{5}{3}x$.

   a. State the domain of j. $\mathbb{R}$

   b. Evaluate $j\left(\frac{27}{10}\right)$. $-\frac{21}{2}$

5. Let $k(x) = -x^2 + 2x + 1$.

   a. State the domain of k. $\mathbb{R}$

   b. Evaluate $k\left(-\frac{5}{6}\right)$. $-\frac{49}{36}$

6. Let $l(x) = \dfrac{2x - 5}{3x + 5}$.

   a. State the domain of l. $x \neq -\frac{5}{3}$

   b. Evaluate $l\left(\frac{15}{2}\right)$. $\frac{4}{11}$

CHAPTER IV

POLYNOMIAL FACTORING AND EQUATIONS

Factor.

1. $40x^9 - 48x^8 - 56x^7 + 32x^6 + 8x^5$

$$= \boxed{8x^5\left(5x^4 - 6x^3 - 7x^2 + 4x + 1\right)}$$

2. $-21x^3y^2z^4 - 14x^2yz^5 + 7xz^6$

$$= \boxed{-7xz^4\left(3x^2y^2 + 2xyz - z^6\right)}$$

3. $\underbrace{2x^4 - 8x^3}\ \underbrace{+ 5x - 20}$

$$= 2x^3(x-4) + 5(x-4)$$

$$= \boxed{(x-4)(2x^3 + 5)}$$

4. $\underbrace{12x^5 - 20x^3y^2}\ \underbrace{-\ 9x^2y + 15y^3}$

$= 4x^3(\underbrace{3x^2 - 5y^2}) - 3y(\underbrace{3x^2 - 5y^2})$

$= \boxed{(3x^2 - 5y^2)(4x^3 - 3y)}$

5. $-2x^{11} - 14x^{10} - 12x^9 - 84x^8$

$= -2x^8(\underbrace{x^3 + 7x^2} + \underbrace{6x + 42})$

$= -2x^8(x^2(\underline{x+7}) + 6(\underline{x+7}))$

$= \boxed{-2x^8(x+7)(x^2+6)}$

6. $45x^{10}y + 10x^9y^3 - 9x^7y^2 - 2x^6y^4$

$= x^6y(\underbrace{45x^4 + 10x^3y^2} \underbrace{-9xy - 2y^3})$

$= x^6y(5x^3(\underline{9x+2y^2}) - y(\underline{9x+2y^2}))$

$= \boxed{x^6y(9x+2y^2)(5x^3-y)}$

Factor.

1. $63x^{10} - 81x^9 + 45x^8 - 36x^7 + 9x^6$

$9x^6(7x^4 - 9x^3 + 5x^2 - 4x + 1)$

2. $-32x^4y^2z^9 - 72x^2yz^{10} + 16xz^{14}$

$-8xz^9(4x^3y^2 + 9xyz - 2z^5)$

3. $14x^5 - 35x^4 + 20x - 50$

$(2x - 5)(7x^4 + 10)$

4. $6x^7 - 27x^4y^2 - 10x^3y + 45y^3$

$(2x^3 - 9y^2)(3x^4 - 5y)$

5. $-4x^8 - 16x^7 - 12x^6 - 48x^5$

$-4x^5(x+4)(x^2+3)$

6. $12x^8y^3 + 30x^7y^5 - 2x^5y^4 - 5x^4y^6$

$x^4y^3(2x+5y^2)(6x^3-y)$

Factor.

1. $x^2 + 13x + 40$

$$= \boxed{(x+8)(x+5)}$$

2. $x^2 - 3xy - 28y^2$

$$= \boxed{(x-7y)(x+4y)}$$

3. $x^2 + 15x - 54$

$$= \boxed{(x+18)(x-3)}$$

4. $-7x^3y + 49x^2y^2 - 42xy^3$

$= -7xy(x^2 - 7xy + 6y^2)$

$= \boxed{-7xy(x-6y)(x-y)}$

5. $5x^4 + 35x^3 - 150x^2$

$= 5x^2(x^2 + 7x - 30)$

$= \boxed{5x(x+10)(x-3)}$

6. $-2x^3y^3 - 40x^2y^4 - 72xy^5$

$= -2xy^3(x^2 + 20xy + 36y^2)$

$= \boxed{-2xy^3(x+18y)(x+2y)}$

Factor.

1. $x^2 + 22x + 72$

$(x+18)(x+4)$

2. $x^2 - 10xy - 24y^2$

$(x-12y)(x+2y)$

3. $x^2 + 9x - 70$

$(x+14)(x-5)$

4. $-5x^3y + 30x^2y^2 - 40xy^3$

$-5xy(x-4y)(x-2y)$

5. $7x^5 + 7x^4 - 294x^3$

$7x^3(x+7)(x-6)$

6. $-3x^4y^4 - 54x^3y^5 - 135x^2y^6$

$-3x^2y^4(x+3y)(x+15y)$

Factor.

1. $9x^2 - 12xy - 5y^2$ $\dfrac{-45}{-15,\,3}$

$= 9x^2 - 15xy + 3xy - 5y^2$

$= 3x(3x - 5y) + y(3x - 5y)$

$= \boxed{(3x - 5y)(3x + y)}$

2. $12x^2 + x - 6$ $\dfrac{-72}{9,\,-8}$

$= 12x^2 + 9x - 8x - 6$

$= 3x(4x + 3) - 2(4x + 3)$

$= \boxed{(4x + 3)(3x - 2)}$

3. $4x^2 - 7xy + 3y^2$ $\dfrac{12}{-4,\,-3}$

$= 4x^2 - 4xy - 3xy + 3y^2$

$= 4x(x - y) - 3y(x - y)$

$= \boxed{(x - y)(4x - 3y)}$

4. $-10x^5 - 19x^4 - 6x^3$

$$= -x^3(10x^2 + 19x + 6)$$

$$= -x^3(10x^2 + 15x + 4x + 6)$$

$$= -x^3(5x(2x+3) + 2(2x+3))$$

$$= \boxed{-x^3(2x+3)(5x+2)}$$

$\dfrac{60}{15, 4}$

5. $12x^7y - 30x^6y^2 - 42x^5y^3$

$$= 6x^5y(2x^2 - 5xy - 7y^2)$$

$$= 6x^5y(2x^2 - 7xy + 2xy - 7y^2)$$

$$= 6x^5y(x(2x-7y) + y(2x-7y))$$

$$= \boxed{6x^5y(2x-7y)(x+y)}$$

$\dfrac{-14}{-7, 2}$

6. $-16x^3 - 22x^2 + 20x$

$$= -2x(8x^2 + 11x - 10)$$

$$= -2x(8x^2 + 16x - 5x - 10)$$

$$= -2x(8x(x+2) - 5(x+2))$$

$$= \boxed{-2x(x+2)(8x-5)}$$

$\dfrac{-80}{16, -5}$

Factor.

1. $12x^2 - 5xy - 3y^2$

$(4x-3y)(3x+y)$

2. $4x^2 + x - 18$

$(4x+9)(x-2)$

3. $9x^2 - 9xy + 2y^2$

$(3x-y)(3x-2y)$

4. $-10x^7 - 21x^6 - 8x^5$

$-x^5(5x+8)(2x+1)$

5. $50x^3y^6 - 105x^2y^7 - 50xy^8$

$5xy^6(2x-5y)(5x+2y)$

6. $-26x^3 - 16x^2 + 10x$

$-2x(x+1)(13x-5)$

Factor.

$(15x)^2 - (14)^2$

1. $225x^2 - 196$

$= \boxed{(15x+14)(15x-14)}$

$(7x^{25})^2 - (11y^{47})^2$

2. $49x^{50} - 121y^{94}$

$= \boxed{(7x^{25}+11y^{47})(7x^{25}-11y^{47})}$

$(9x^2)^2 - (1)^2$

3. $81x^4 - 1$ $(3x)^2 - (1)^2$

$= (9x^2+1)(9x^2-1)$

$= \boxed{(9x^2+1)(3x+1)(3x-1)}$

4. $-20x^{23}y^4 + 125xy^{70}$

$(2x^{11})^2 - (5y^{37})^2$

$= -5xy^4 \left(4x^{22} - 25y^{66}\right)$

$= \boxed{-5xy^4 \left(2x^{11} + 5y^{33}\right)\left(2x^{11} - 5y^{33}\right)}$

5. $360x^{15} - 720x^{14} - 10x + 20$

$= 10 \left(36x^{15} - 72x^{14} - x + 2\right)$

$= 10 \left(36x^{14}(x-2) - 1(x-2)\right)$

$(6x^7)^2 - (1)^2$

$= 10(x-2)\left(36x^{14} - 1\right)$

$= \boxed{10(x-2)(6x^7+1)(6x^7-1)}$

6. $-4x^{12}y - 2x^{11}y^2 + 64x^4y^5 + 32x^3y^6$

$= -2x^3y \left(2x^9 + x^8 y - 32xy^4 - 16y^5\right)$

$= -2x^3y \left(x^8(2x+y) - 16y^4(2x+y)\right)$

$(x^4)^2 - (4y^2)^2$

$= -2x^3y (2x+y)\left(x^8 - 16y^4\right)$

$(x^2)^2 - (2y)^2$

$= -2x^3y (2x+y)\left(x^4 + 4y^2\right)\left(x^4 - 4y^2\right)$

$= \boxed{-2x^3y (2x+y)\left(x^4 + 4y^2\right)\left(x^2 + 2y\right)\left(x^2 - 2y\right)}$

Factor.

1. $196x^2 - 169$

$$(14x+13)(14x-13)$$

2. $121x^{58} - 225y^{92}$

$$(11x^{29} + 15y^{46})(11x^{29} - 15y^{46})$$

3. $16x^{12} - 1$

$$(4x^6+1)(2x^3+1)(2x^3-1)$$

4. $-36x^{31}y^5 + 400xy^{59}$

$-4xy^5(3x^{15}+10y^{27})(3x^{15}-10y^{27})$

5. $500x^{19} - 750x^{18} - 20x + 30$

$10(2x-3)(5x^9+1)(5x^9-1)$

6. $-6x^{12}y - 6x^{11}y^2 + 486x^4y^5 + 486x^3y^6$

$-6x^3y(x+y)(x^4+9y^2)(x^2+3y)(x^2-3y)$

Factor.

1. $-45x^4 - 27x^3 - 80x^2 - 48x$

$$= -x\left(45x^3 + 27x^2 + 80x + 48\right)$$

$$= -x\left(9x^2(5x+3) + 16(5x+3)\right)$$

$$= \boxed{-x(5x+3)(9x^2+16)}$$

2. $8x^2 + 10xy - 25y^2$

$$\frac{-200}{20, -10}$$

$$= 8x^2 + 20xy - 10xy - 25y^2$$

$$= 4x(2x+5y) - 5y(2x+5y)$$

$$= \boxed{(2x+5y)(4x-5y)}$$

3. $4x^6 - 8x^5 - 100x^4 + 200x^3$

$$= 4x^3\left(x^3 - 2x^2 - 25x + 50\right)$$

$$= 4x^3\left(x^2(x-2) - 25(x-2)\right)$$

$$= 4x^3(x-2)(x^2-25)$$

$$= \boxed{4x^3(x-2)(x+5)(x-5)}$$

4. $-5x^{10} + 100x^9 - 500x^8$

$= -5x^8 (x^2 - 20x + 100)$

$= \boxed{-5x^8 (x-10)(x-10)}$

5. $160x^6y^3 + 32x^5y^4 - 810x^2y^7 - 162xy^8$

$= 2xy^3 (80x^5 + 16x^4y - 405xy^4 - 81y^5)$

$= 2xy^3 (16x^4 (5x+y) - 81y^4 (5x+y))$

$= 2xy^3 (5x+y)(16x^4 - 81y^4)$

$= 2xy^3 (5x+y)(4x^2 + 9y^2)(4x^2 - 9y^2)$

$= \boxed{2xy^3 (5x+y)(4x^2 + 9y^2)(2x+3y)(2x-3y)}$

6. $-54x^2y^2 + 45xy^3 + 36y^4$

$\dfrac{-24}{-8, 3}$

$= -9y^2 (6x^2 - 5xy - 4y^2)$

$= -9y^2 (6x^2 - 8xy + 3xy - 4y^2)$

$= -9y^2 (2x(3x-4y) + y(3x-4y))$

$= \boxed{-9y^2 (3x-4y)(2x+y)}$

Factor.

1. $-243x^4 - 162x^3 - 75x^2 - 50x$

$-x(3x+2)(81x^2+25)$

2. $20x^2 + 7xy - 6y^2$

$(4x+3y)(5x-2y)$

3. $3x^7 - 15x^6 - 48x^5 + 240x^4$

$3x^4(x-5)(x+4)(x-4)$

4. $-4x^8 + 120x^7 - 900x^6$

$-4x^6 (x-15)(x-15)$

5. $324x^8y^4 + 486x^7y^5 - 4x^4y^8 - 6x^3y^9$

$2x^3y^4 (2x+3y)(9x^2+y^2)(3x+y)(3x-y)$

6. $-50x^2y^3 + 15xy^4 + 20y^5$

$-5y^3 (5x-4y)(2x+y)$

Solve.

1. $49x^2 - 16 = 0$

$$(7x+4)(7x-4) = 0$$

$$\boxed{x = -\tfrac{4}{7}, \ \tfrac{4}{7}}$$

2. $18x^2 + 17x + 4 = 0$ $\dfrac{72}{9,8}$

$$18x^2 + 9x + 8x + 4 = 0$$

$$9x(2x+1) + 4(2x+1) = 0$$

$$(2x+1)(9x+4) = 0$$

$$\boxed{x = -\tfrac{1}{2}, \ -\tfrac{4}{9}}$$

3. $36x^2 = 4x$

$$36x^2 - 4x = 0$$

$$4x(9x-1) = 0$$

$$\boxed{x = 0, \ \tfrac{1}{9}}$$

4. $(4x - 7)(x + 2) = -9$

$4x^2 + 8x - 7x - 14 = -9$

$4x^2 + x - 14 = -9$ $\dfrac{-20}{5, -4}$

$4x^2 + x - 5 = 0$

$4x^2 + 5x - 4x - 5 = 0$

$x(4x + 5) - 1(4x + 5) = 0$

$(4x + 5)(x - 1) = 0$

$\boxed{x = -\dfrac{5}{4}, 1}$

5. $\left(\dfrac{3}{40}x^2 + \dfrac{12}{5}\right) = \left(\dfrac{9}{10}x\right)$

$3x^2 + 96 = 36x$

$3x^2 - 36x + 96 = 0$

$3(x^2 - 12x + 32) = 0$

$3(x - 8)(x - 4) = 0$

$\boxed{x = 8, 4}$

6. $(2x + 5)(2x - 9) - 16x = 3x(x - 8) + 99$

$4x^2 - 18x + 10x - 45 - 16x = 3x^2 - 24x + 99$

$4x^2 - 24x - 45 = 3x^2 - 24x + 99$

$x^2 - 144 = 0$

$(x + 12)(x - 12) = 0$

$\boxed{x = -12, 12}$

Solve.

1. $225x^2 - 49 = 0$

$x = \frac{-7}{15}, \frac{7}{15}$

2. $15x^2 + 23x + 4 = 0$

$x = -\frac{4}{3}, -\frac{1}{5}$

3. $100x^2 = 25x$

$x = 0, \frac{1}{4}$

4. $(4x - 9)(x + 2) = -4$

$x = 2, -\frac{7}{4}$

5. $\frac{2}{45}x^2 + \frac{14}{9} = \frac{8}{15}x$

$x = 5, 7$

6. $(8x - 3)(4x + 5) + 23 = 9 - 4x(x - 7)$

$x = -\frac{1}{6}, \frac{1}{6}$

CHAPTER V

RATIONAL EXPRESSIONS AND EQUATIONS

1. Let $f(x) = \dfrac{x^2 - 9x + 7}{24x^2 - 54x}$. State the domain of f.

$$24x^2 - 54x = 0$$

$$6x(4x - 9) = 0$$

$$\boxed{x \neq 0, \, 9/4}$$

Simplify.

2. $\dfrac{\overset{①}{9x^3 - 18x^2y - 16xy^2 + 32y^3}}{\underset{②}{18x^3y + 48x^2y^2 + 32xy^3}} = \dfrac{(x-2y)(3x+4y)(3x-4y)}{2xy(3x+4y)(3x+4y)} = \boxed{\dfrac{(x-2y)(3x-4y)}{2xy(3x+4y)}}$

① $9x^2(x-2y) - 16y^2(x-2y)$
$(x-2y)(9x^2 - 16y^2)$
$(x-2y)(3x+4y)(3x-4y)$

② $2xy(9x^2 + 24xy + 16y^2)$ $\dfrac{144}{12,12}$
$2xy(9x^2 + 12xy + 12xy + 16y^2)$
$2xy(3x(3x+4y) + 4y(3x+4y))$
$2xy(3x+4y)(3x+4y)$

3. $\dfrac{4xy^2}{2x^3y^3 + 10y^4} \div \dfrac{24x^5 - 6x^3y^2}{6x^6y - 3x^5y^2 + 30x^3y^2 - 15x^2y^3}$

$= \dfrac{\overset{①}{4xy^2}}{\underset{②}{2x^3y^3 + 10y^4}} \cdot \dfrac{\overset{③}{6x^6y - 3x^5y^2 + 30x^3y^2 - 15x^2y^3}}{\underset{④}{24x^5 - 6x^3y^2}} = \dfrac{\cancel{4xy^2} \cdot \cancel{3x^2y}(2x-y)(x^3+5y)}{\cancel{2y^3}(x^3+5y) \cdot \cancel{6x^3}(2x+y)(2x-y)} = \boxed{\dfrac{1}{2x+y}}$

① $4xy^2$

② $2y^3(x^3+5y)$

③ $3x^2y(2x^4 - x^3y + 10xy - 5y^2)$
$3x^2y(x^3(2x-y) + 5y(2x-y))$
$3x^2y(2x-y)(x^3+5y)$

④ $6x^3(4x^2 - y^2)$
$6x^3(2x+y)(2x-y)$

4. $\dfrac{\overset{①}{x^2+5x-36}}{\underset{②}{x^2+16x+63}} \cdot \dfrac{\overset{③}{x^2-7x-60}}{\underset{④}{x^2-16x+48}} = \dfrac{(x+9)(x-4)}{(x+9)(x+7)} \cdot \dfrac{(x-12)(x+5)}{(x-12)(x-4)} = \boxed{\dfrac{x+5}{x+7}}$

① $(x+9)(x-4)$ ③ $(x-12)(x+5)$

② $(x+9)(x+7)$ ④ $(x-12)(x-4)$

5. $\dfrac{2+2x-2x^2-2x^3}{4x^2+8x+4} \div \dfrac{-2x^2+4x-2}{3x^2+2x-5}$

$= \dfrac{\overset{①}{-2x^3-2x^2+2x+2}}{\underset{②}{4x^2+8x+4}} \cdot \dfrac{\overset{③}{3x^2+2x-5}}{\underset{④}{-2x^2+4x-2}} = \dfrac{-2(x+1)(x+1)(x-1)}{4(x+1)(x+1)} \cdot \dfrac{(3x+5)(x-1)}{-2(x+1)(x-1)} = \boxed{\dfrac{3x+5}{4}}$

$\overset{-15}{5,-3}$

① $-2(x^3+x^2-x-1)$ ② $4(x^2+2x+1)$ ③ $3x^2+5x-3x-5$ ④ $-2(x^2-2x+1)$

$\quad -2(x^2(x+1)-1(x+1))$ $\quad 4(x+1)(x+1)$ $\quad x(3x+5)-1(3x+5)$ $\quad -2(x-1)(x-1)$

$\quad -2(x+1)(x^2-1)$ $\quad (3x+5)(x-1)$

$\quad -2(x+1)(x+1)(x-1)$

6. $\dfrac{\overset{①}{30x^2+130xy-100y^2}}{\underset{②}{10x^2+48xy-10y^2}} \cdot \dfrac{\overset{③}{7y^2-21xy-70x^2}}{\underset{④}{30x^2-5xy-10y^2}} = \dfrac{\overset{7}{10}(x+5y)(3x-2y)}{2(x+5y)(5x-y)} \cdot \dfrac{-7(2x+y)(5x-y)}{5(3x-2y)(2x+y)} = \boxed{-7}$

$\overset{-30}{15,-2}$ $\overset{-25}{25,-1}$ $\overset{-10}{5,-2}$

① $10(3x^2+13xy-10y^2)$ ② $2(5x^2+24xy-5y^2)$ ③ $-7(10x^2+3xy-y^2)$

$\quad 10(3x^2+15xy-2xy-10y^2)$ $\quad 2(5x^2+25xy-1xy-5y^2)$ $\quad -7(10x^2+5xy-2xy-y^2)$

$\quad 10(3x(x+5y)-2y(x+5y))$ $\quad 2(5x(x+5y)-y(x+5y))$ $\quad -7(5x(2x+y)-y(2x+y))$

$\quad 10(x+5y)(3x-2y)$ $\quad 2(x+5y)(5x-y)$ $\quad -7(2x+y)(5x-y)$

④ $5(6x^2-xy-2y^2)$ $\overset{-12}{-4,3}$

$\quad 5(6x^2-4xy+3xy-2y^2)$

$\quad 5(2x(3x-2y)+y(3x-2y))$

$\quad 5(3x-2y)(2x+y)$

1. Let $f(x) = \dfrac{x^2 + 8x - 15}{45x^2 - 27x}$. State the domain of f.

$x \neq 0, \frac{3}{5}$

Simplify.

2. $\dfrac{16x^3 + 80x^2y - 9xy^2 - 45y^3}{80x^3y - 120x^2y^2 + 45xy^3}$

$\dfrac{(x+5y)(4x+3y)}{5xy(4x-3y)}$

3. $\dfrac{6x^2y}{6x^3y^2 + 3y^3} \div \dfrac{10x^5 - 40x^3y^2}{10x^5y^2 - 20x^4y^3 + 5x^2y^3 - 10xy^4}$

$\dfrac{y}{x+2y}$

4. $\dfrac{x^2 + 8x - 20}{x^2 + 19x + 90} \cdot \dfrac{x^2 - 12x - 45}{x^2 - 17x + 30}$

$\dfrac{x+3}{x+9}$

5. $\dfrac{-3 + 3x + 3x^2 - 3x^3}{7x^2 - 14x + 7} \div \dfrac{-3x^2 - 6x - 3}{9x^2 + x - 8}$

$\dfrac{9x-8}{7}$

6. $\dfrac{20x^2 + 50xy - 120y^2}{20x^2 + 75xy - 20y^2} \cdot \dfrac{12y^2 + 12xy - 240x^2}{40x^2 - 52xy - 12y^2}$

-6

Simplify.

1. $\dfrac{89x^3 + 73x}{200x^3 + 300x^2 - 2x - 3} + \dfrac{11x^3 - 74x}{200x^3 + 300x^2 - 2x - 3}$ $=$ $\dfrac{89x^3 + 73x + 11x^3 - 74x}{(89x^3 + 73x) + (11x^3 - 74x)}$

$= \dfrac{(89x^3 + 73x) + (11x^3 - 74x)}{200x^3 + 300x^2 - 2x - 3} =$

① $x(100x^2 - 1)$
$x(10x+1)(10x-1)$

② $100x^2(2x+3) - 1(2x+3)$
$(2x+3)(100x^2 - 1)$
$(2x+3)(10x+1)(10x-1)$

$= \dfrac{① \ 100x^3 - x}{② \ 200x^3 + 300x^2 - 2x - 3} = \dfrac{x(10x+1)(10x+1)}{(2x+3)(10x+1)(10x-1)} =$

$= \boxed{\dfrac{x}{2x+3}}$

2. $\dfrac{10y^2 + 5xy - 2x^2}{x^2 + 18xy + 81y^2} - \dfrac{3x^2 + 40xy - 80y^2}{x^2 + 18xy + 81y^2}$ $=$ $\dfrac{10y^2 + 5xy - 2x^2 - 3x^2 - 40xy + 80y^2}{(10y^2 + 5xy - 2x^2) - (3x^2 + 40xy - 80y^2)}$

$= \dfrac{(10y^2 + 5xy - 2x^2) - (3x^2 + 40xy - 80y^2)}{x^2 + 18xy + 81y^2} =$

① $-5(x^2 + 7xy - 18y^2)$
$-5(x+9y)(x-2y)$

② $(x+9y)(x+9y)$

$= \dfrac{① \ -5x^2 - 35xy + 90y^2}{② \ x^2 + 18xy + 81y^2} = \dfrac{-5(x+9y)(x-2y)}{(x+9y)(x+9y)} =$

$= \boxed{\dfrac{-5(x-2y)}{x+9y}}$

3. $\dfrac{4x^2 - x - 13}{6x^2 - 11x - 10} + \dfrac{8 + 3x - 4x^2}{6x^2 - 11x - 10}$ $=$ $\dfrac{4x^2 - x - 13 + 8 + 3x - 4x^2}{(4x^2 - x - 13) + (8 + 3x - 4x^2)}$

$= \dfrac{(4x^2 - x - 13) + (8 + 3x - 4x^2)}{6x^2 - 11x - 10} = \dfrac{2x - 5}{① \ 6x^2 - 11x - 10} =$

① $6x^2 - 15x + 4x - 10 \qquad \dfrac{-60}{-15, 4}$
$3x(2x-5) + 2(2x-5)$
$(2x-5)(3x+2)$

$= \dfrac{2x - 5 \quad 1}{(2x-5)(3x+2)}$

$= \boxed{\dfrac{1}{3x+2}}$

4. $\dfrac{x^3}{x^2y - xy^2} - \dfrac{x^2y - xy^2 + y^3}{x^2y - xy^2} = \dfrac{x^3 - (x^2y - xy^2 + y^3)}{x^2y - xy^2} = $

$$\overset{x^3 - x^2y + xy^2 - y^3}{\dfrac{x^3 - (x^2y - xy^2 + y^3)}{x^2y - xy^2}}$$

$$\overset{①}{=} \dfrac{x^3 - x^2y + xy^2 - y^3}{x^2y - xy^2} = $$

②

① $x^2(x-y) + y^2(x-y)$

$(x-y)(x^2+y^2)$

② $xy(x-y)$

$$= \dfrac{(x-y)(x^2+y^2)}{xy(x-y)} = \boxed{\dfrac{x^2+y^2}{xy}}$$

5. $\dfrac{17x^2 - 2x + 4}{5x - 4} + \dfrac{12x^2 + 7x}{4 - 5x} = \dfrac{17x^2 - 2x + 4}{5x - 4} - \dfrac{12x^2 + 7x}{5x - 4} = $

$$\overset{17x^2 - 2x + 4 - 12x^2 - 7x}{\dfrac{(17x^2 - 2x + 4) - (12x^2 + 7x)}{5x - 4}} = $$

① $5x^2 - 4x - 5x + 4$ $\quad \overset{\frac{20}{-4,-5}}{}$

$x(5x-4) - 1(5x-4)$

$(5x-4)(x-1)$

$$= \overset{①}{\dfrac{5x^2 - 9x + 4}{5x - 4}} = \dfrac{(5x-4)(x-1)}{5x-4} = \boxed{x-1}$$

6. $\dfrac{6x^5y^3 - 5x^4y^4}{32x^5y^4 - 98x^3y^6} - \dfrac{2x^5y^3 - 9x^4y^4}{98x^3y^6 - 32x^5y^4} = \dfrac{6x^5y^3 - 5x^4y^4}{32x^5y^4 - 98x^3y^6} + \dfrac{2x^5y^3 - 9x^4y^4}{32x^5y^4 - 98x^3y^6} = $

① $2x^4y^3(4x - 7y)$

② $2x^3y^4(16x^2 - 49y^2)$

$\quad 2x^3y^4(4x+7y)(4x-7y)$

$$= \overset{6x^5y^3 - 5x^4y^4 + 2x^5y^3 - 9x^4y^4}{\dfrac{(6x^5y^3 - 5x^4y^4) + (2x^5y^3 - 9x^4y^4)}{32x^5y^4 - 98x^3y^6}} = $$

$$= \dfrac{\overset{①}{8x^5y^3 - 14x^4y^4}}{\underset{②}{32x^5y^4 - 98x^3y^6}} = \dfrac{2x^4y^3(4x-7y)}{2x^3y^4(4x+7y)(4x-7y)}$$

$$= \boxed{\dfrac{x}{y(4x+7y)}}$$

Simplify.

1. $\dfrac{17x^3 - 57x}{98x^3 + 49x^2 - 200x - 100} + \dfrac{32x^3 - 43x}{98x^3 + 49x^2 - 200x - 100}$

$$\dfrac{x}{2x+1}$$

2. $\dfrac{25y^2 + 8xy - x^2}{x^2 + 16xy + 64y^2} - \dfrac{4x^2 + 33xy - 95y^2}{x^2 + 16xy + 64y^2}$

$$\dfrac{-5(x-3y)}{x+8y}$$

3. $\dfrac{9x^2 - 17x - 21}{14x^2 + 3x - 5} + \dfrac{20 + 19x - 9x^2}{14x^2 + 3x - 5}$

$$\dfrac{1}{7x+5}$$

4. $\dfrac{x^7 + x^5 y^2}{x^7 y^2 - x^2 y^5} - \dfrac{x^2 y^3 + y^5}{x^7 y^2 - x^2 y^5}$

$\dfrac{x^2 + y^2}{x^2 y^2}$

5. $\dfrac{31x^2 + 2x}{9x - 2} + \dfrac{22x^2 - 5x + 2}{2 - 9x}$

$x + 1$

6. $\dfrac{5x^9 y^9 + 21x^8 y^{10}}{72x^{11} y^8 - 50x^9 y^{10}} - \dfrac{7x^9 y^9 - 11x^8 y^{10}}{50x^9 y^{10} - 72x^{11} y^8}$

$\dfrac{y}{x(6x - 5y)}$

Simplify.

1. $\dfrac{2}{xy^2} + \dfrac{1}{x^2y} = \dfrac{2x}{x \cdot xy^2} + \dfrac{1 \cdot y}{x^2y \cdot y} = \dfrac{2x}{x^2y^2} + \dfrac{y}{x^2y^2} = \boxed{\dfrac{2x+y}{x^2y^2}}$

2. $\dfrac{4x}{(3x+5)} - \dfrac{4x}{(3x-5)} = \dfrac{4x(3x-5)}{(3x+5)(3x-5)} - \dfrac{4x(3x+5)}{(3x-5)(3x+5)} = \dfrac{\overset{12x^2-20x \,-\, 12x^2 - 20x}{\overbrace{4x(3x-5)-4x(3x+5)}}}{(3x+5)(3x-5)} =$

$= \boxed{\dfrac{-40x}{(3x+5)(3x-5)}}$

3. $\dfrac{(2x+3y)}{(2x-3y)} + \dfrac{(2x-3y)}{(2x+3y)} = \dfrac{(2x+3y)(2x+3y)}{(2x-3y)(2x+3y)} + \dfrac{(2x-3y)(2x-3y)}{(2x+3y)(2x-3y)} =$

$= \dfrac{\overset{4x^2+6xy+6xy+9y^2 \,+\, 4x^2-6xy-6xy+9y^2}{\overbrace{(2x+3y)(2x+3y) + (2x-3y)(2x-3y)}}}{(2x-3y)(2x+3y)} = \dfrac{\overset{①}{8x^2} + 18y^2}{(2x-3y)(2x+3y)} =$

$= \boxed{\dfrac{2(4x^2+9y^2)}{(2x-3y)(2x+3y)}}$

① $2(4x^2+9y^2)$

4. $\dfrac{x+4}{x^2+8x} - \dfrac{x}{x^2-64}$ $= \dfrac{(x+4)}{x(x+8)} - \dfrac{x}{(x+8)(x-8)} = \dfrac{(x+4)(x-8)}{x(x+8)(x-8)} - \dfrac{x \cdot x}{x(x+8)(x-8)} =$

$$= \dfrac{\overset{x^2-8x+4x-32-x^2}{(x+4)(x-8)} - x^2}{x(x+8)(x-8)} = \dfrac{-4x-32}{x(x+8)(x-8)} = \dfrac{-4(x+8)}{x(x+8)(x-8)}$$

$$= \boxed{\dfrac{-4}{x(x-8)}}$$

5. $\dfrac{-4x^2-11xy+8y^2}{25x^3-50x^2y-16xy^2+32y^3} + \dfrac{x+3y}{5x^2-6xy-8y^2} = \dfrac{-4x^2-11xy+8y^2}{(x-2y)(5x+4y)(5x-4y)} + \dfrac{(x+3y)(5x-4y)}{(x-2y)(5x+4y)(5x-4y)} =$

$25x^2(x-2y) -16y^2(x-2y)$ $\qquad$ $5x^2-10xy+4xy-8y^2$

$(x-2y)(25x^2-16y^2)$ $\qquad$ $5x(x-2y)+4y(x-2y)$ $\qquad \dfrac{-40}{-10,4}$

$(x-2y)(5x+4y)(5x-4y)$ $\qquad$ $(x-2y)(5x+4y)$

$$= \dfrac{\overset{-4x^2-11xy+8y^2+5x^2-4xy+15xy-12y^2}{-4x^2-11xy+8y^2 + (x+3y)(5x-4y)}}{(x-2y)(5x+4y)(5x-4y)} =$$

① $(x+2y)(x-2y)$ $\qquad$ $\boxed{\dfrac{x+2y}{(5x+4y)(5x-4y)}}$

$$= \dfrac{\overset{①}{x^2-4y^2}}{(x-2y)(5x+4y)(5x-4y)} = \dfrac{(x+2y)(x-2y)}{(x-2y)(5x+4y)(5x-4y)}$$

6. $\dfrac{2x^2+11x+64}{x^2+3x-40} - \dfrac{x}{x+8}$ $= \dfrac{2x^2+11x+64}{(x+8)(x-5)} - \dfrac{x(x-5)}{(x+8)(x-5)} = \dfrac{2x^2+11x+64-x(x-5)}{(x+8)(x-5)} =$

$(x+8)(x-5)$

$$= \dfrac{2x^2+11x+64-x^2+5x}{(x+8)(x-5)} = \dfrac{\overset{①}{x^2+16x+64}}{(x+8)(x-5)} = \dfrac{(x+8)(x+8)}{(x+8)(x-5)} =$$

$$= \boxed{\dfrac{x+8}{x-5}}$$

① $(x+8)(x+8)$

Simplify.

1. $\dfrac{3}{xy^3} + \dfrac{2}{x^2y^2}$

$\dfrac{3x+2y}{x^2y^3}$

2. $\dfrac{10x}{8x-3} - \dfrac{10x}{8x+3}$

$\dfrac{60x}{(8x-3)(8x+3)}$

3. $\dfrac{9x-y}{9x+y} + \dfrac{9x+y}{9x-y}$

$\dfrac{2(81x^2+y^2)}{(9x+y)(9x-y)}$

4. $\dfrac{x+6}{x^2+12x} - \dfrac{x}{x^2-144}$

$\dfrac{-6}{x-12}$

5. $\dfrac{-15x^2+7xy+6y^2}{27x^3-9x^2y-75xy^2+25y^3} + \dfrac{2x+y}{9x^2+12xy-5y^2}$

$\dfrac{-(3x+y)}{(3x+5y)(3x-5y)}$

6. $\dfrac{2x^2-7x+36}{x^2-x-30} - \dfrac{x}{x-6}$

$\dfrac{x-6}{x+5}$

Simplify.

1. $\dfrac{\left(8 - \dfrac{6}{x}\right)}{\left(12 - \dfrac{9}{x}\right)} \cdot \dfrac{x}{x} = \dfrac{8x - 6}{12x - 9} = \dfrac{2(4x - 3)}{3(4x - 3)} = \boxed{\dfrac{2}{3}}$

2. $\dfrac{\left(\dfrac{3}{xy^2} + \dfrac{2}{x^2y}\right)}{\left(\dfrac{9}{y^2} - \dfrac{4}{x^2}\right)} \cdot \dfrac{x^2y^2}{x^2y^2} = \dfrac{3x + 2y}{9x^2 - 4y^2} = \dfrac{3x + 2y}{(3x+2y)(3x-2y)} = \boxed{\dfrac{1}{3x - 2y}}$

3. $\dfrac{\left(2 - \dfrac{3}{x} - \dfrac{20}{x^2}\right)}{\left(3 - \dfrac{13}{x} + \dfrac{4}{x^2}\right)} \cdot \dfrac{x^2}{x^2} = \dfrac{②\ 2x^2 - 3x - 20}{②\ 3x^2 - 13x + 4} = \dfrac{(x-4)(2x+5)}{(x-4)(3x-1)} = \boxed{\dfrac{2x + 5}{3x - 1}}$

① $2x^2 - 8x + 5x - 20 \qquad \dfrac{-40}{-8, 5}$

$2x(x-4) + 5(x-4)$

$(x-4)(2x+5)$

② $3x^2 - 12x - 1x + 4 \qquad \dfrac{12}{-12, -1}$

$3x(x-4) - 1(x-4)$

$(x-4)(3x-1)$

4. $\dfrac{\left(\dfrac{1}{y^2}+\dfrac{3}{xy}-\dfrac{4}{x^2}\right)}{\left(\dfrac{1}{y^2}+\dfrac{5}{xy}+\dfrac{4}{x^2}\right)}\cdot\dfrac{x^2y^2}{x^2y^2}=\dfrac{x^2+3xy-4y^2}{x^2+5xy+4y^2}=\dfrac{(x+4y)(x-y)}{(x+4y)(x+y)}=\boxed{\dfrac{x-y}{x+y}}$

5. $\dfrac{\left(\dfrac{20}{x^2}+\dfrac{45}{x^4}\right)}{\left(\dfrac{16}{x^2}-\dfrac{81}{x^6}\right)}\cdot\dfrac{x^6}{x^6}=\dfrac{\overset{①}{20x^4+45x^2}}{\underset{②}{16x^4-81}}=\dfrac{5x^2(4x^2+9)}{(4x^2+9)(2x+3)(2x-3)}=\boxed{\dfrac{5x^2}{(2x+3)(2x-3)}}$

① $5x^2(4x^2+9)$

② $(4x^2+9)(4x^2-9)$

$\quad(4x^2+9)(2x+3)(2x-3)$

6. $\dfrac{\left(\dfrac{1}{y^3}+\dfrac{1}{xy^2}-\dfrac{1}{x^2y}-\dfrac{1}{x^3}\right)}{\left(\dfrac{1}{y^3}+\dfrac{1}{xy^2}+\dfrac{1}{x^2y}+\dfrac{1}{x^3}\right)}\cdot\dfrac{x^3y^3}{x^3y^3}=\dfrac{\overset{①}{x^3+x^2y-xy^2-y^3}}{\underset{②}{x^3+x^2y+xy^2+y^3}}=\dfrac{(x+y)(x+y)(x-y)}{(x+y)(x^2+y^2)}$

$=\boxed{\dfrac{(x+y)(x-y)}{x^2+y^2}}$

① $x^2(x+y)-y^2(x+y)$

$\quad(x+y)(x^2-y^2)$

$\quad(x+y)(x+y)(x-y)$

② $x^2(x+y)+y^2(x+y)$

$\quad(x+y)(x^2+y^2)$

Simplify.

1. $\dfrac{15 - \dfrac{20}{x}}{6 - \dfrac{8}{x}}$

$\dfrac{5}{2}$

2. $\dfrac{\dfrac{7}{x^4 y^4} + \dfrac{9}{x^5 y^3}}{\dfrac{49}{x^3 y^4} - \dfrac{81}{x^5 y^2}}$

$\dfrac{1}{7x - 9y}$

3. $\dfrac{6 + \dfrac{29}{x} - \dfrac{5}{x^2}}{5 + \dfrac{27}{x} + \dfrac{10}{x^2}}$

$\dfrac{6x - 1}{5x + 2}$

4. $\dfrac{\dfrac{1}{y^2} - \dfrac{7}{xy} - \dfrac{8}{x^2}}{\dfrac{1}{y^2} - \dfrac{9}{xy} + \dfrac{8}{x^2}}$

$\dfrac{x+y}{x-y}$

5. $\dfrac{\dfrac{54}{x^4} + \dfrac{24}{x^6}}{\dfrac{81}{x^6} - \dfrac{16}{x^{10}}}$

$\dfrac{6x^4}{(3x+2)(3x-2)}$

6. $\dfrac{\dfrac{1}{xy^7} - \dfrac{1}{x^2y^6} - \dfrac{1}{x^5y^3} + \dfrac{1}{x^6y^2}}{\dfrac{1}{x^3y^7} + \dfrac{1}{x^4y^6} + \dfrac{1}{x^5y^5} + \dfrac{1}{x^6y^4}}$

$(x-y)^2$

Solve.

1. $\dfrac{x+6}{2x+3} \times \dfrac{2x-3}{x}$ $\left(x \neq -\frac{3}{2}, 0\right)$

$x(x+6) = (2x-3)(2x+3)$

$x^2 + 6x = 4x^2 + 6x - 6x - 9$

$x^2 + 6x = 4x^2 - 9$

$0 = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$0 = 3(x-3)(x+1)$ $\boxed{x = 3, -1}$

2. $(x+4)\left(7 - \dfrac{3x}{x+4}\right) = \left(\dfrac{2-9x}{x+4}\right)(x+4)$ $(x \neq -4)$

$7(x+4) - 3x = (2-9x)$

$7x + 28 - 3x = 2 - 9x$

$4x + 28 = 2 - 9x$

$\dfrac{13x}{13} = \dfrac{-26}{13}$

$\boxed{x = -2}$

3. $(3x-2)(4x+3)\left(\dfrac{5x+3}{(3x-2)}\right) = \left(\dfrac{10x^2+23x+18}{12x^2+x-6} - \dfrac{2x}{(4x+3)}\right)(3x-2)(4x+3)$ $\left(x \neq \frac{2}{3}, -\frac{3}{4}\right)$

$(5x+3)(4x+3) = (10x^2 + 23x + 18) - 2x(3x-2)$

$20x^2 + 15x + 12x + 9 = 10x^2 + 23x + 18 - 6x^2 + 4x$

$20x^2 + 27x + 9 = 4x^2 + 27x + 18$

$16x^2 - 9 = 0$

$(4x+3)(4x-3) = 0$

$\boxed{x = -\frac{3}{4}, \frac{3}{4}}$

$(x+7)(x+2)$

$4\left(\dfrac{5x}{x+7} - 3\right) = \left(\dfrac{-28x-57}{x^2+9x+14}\right)(x+7)(x+2)$ $(x \neq -7, -2)$

$(x+7)(x+2)$

$5x(x+2) - 3(x+7)(x+2) = (-28x-57)$ $\longrightarrow$ $(2x+5)(x+3) = 0$

$5x(x+2) - 3(x^2+9x+14) = -28x-57$

$5x^2+10x-3x^2-27x-42 = -28x-57$ $\boxed{x = -\tfrac{5}{2}, -3}$

$2x^2-17x-42 = -28x-57$

$2x^2+11x+15 = 0$ $\quad \dfrac{30}{5,6}$

$\underline{2x^2+5x}+\underline{6x+15} = 0$

$x(2x+5) + 3(2x+5) = 0$

$(25x^2+16)(4x-1)$

$5.\left(\dfrac{25x^3+8x^2-25x+3}{100x^3-25x^2+64x-16}\right) = \left(\dfrac{x-3}{25x^2+16} + \dfrac{x}{4x-1}\right)(25x^2+16)(4x-1)$ $(x \neq \tfrac{1}{4})$

$(25x^2+16)(4x-1)$

$(25x^3+8x^2-25x+3) = (x-3)(4x-1) + x(25x^2+16)$

$25x^3+8x^2-25x+3 = 4x^2-x-12x+3 + 25x^3+16x$

$\cancel{25x^3}+8x^2-25x\cancel{+3} = \cancel{25x^3}+4x^2+3x\cancel{+3}$

$4x^2-28x = 0$

$4x(x-7) = 0$

$\boxed{x = 0, 7}$

$(3x+5)(3x-5)$

$6.\left(\dfrac{3x+2}{3x+5}\right) = \left(\dfrac{9x}{3x-5} - 2\right)(3x+5)(3x-5)$ $\left(x \neq -\tfrac{5}{3}, \tfrac{5}{3}\right)$

$(3x+2)(3x-5) = 9x(3x+5) - 2(3x+5)(3x-5)$

$(3x+2)(3x-5) = 9x(3x+5) - 2(9x^2-25)$

$9x^2-15x+6x-10 = 27x^2+45x-18x^2+50$

$\cancel{9x^2}-9x-10 = \cancel{9x^2}+45x+50$

$\dfrac{-\cancel{54}x}{-54} = \dfrac{60}{-54}$

$\boxed{x = -\tfrac{10}{9}}$

Solve.

1. $\dfrac{x}{3x-2} = \dfrac{3x+2}{x-4}$

$x = \frac{1}{2}, -1$

2. $8 - \dfrac{5x}{x+9} = \dfrac{2-11x}{x+9}$

$x = -5$

3. $\dfrac{4x-3}{7x+2} = \dfrac{17x^2 - 30x + 40}{56x^2 - 19x - 10} - \dfrac{7x}{8x-5}$

$x = -\frac{5}{8}$

4. $\dfrac{8x}{x+6} - 5 = \dfrac{-35x - 114}{x^2 + 10x + 24}$

$x = \frac{1}{3}$

5. $\dfrac{48x^3 + 17x^2 + 195x + 1}{32x^3 - 16x^2 + 162x - 81} = \dfrac{x-1}{16x^2 + 81} + \dfrac{3x}{2x-1}$

$x = 0, 3$

6. $\dfrac{4x+3}{4x+7} = \dfrac{8x}{4x-7} - 1$

$x = \dfrac{-35}{36}$

CHAPTER VI

RADICAL EXPRESSIONS AND EQUATIONS

Simplify. Assume all variables are nonnegative.

1. $\sqrt{196x^{26}}$ = $\boxed{14x^{13}}$

2. $\sqrt[3]{64x^{30}}$ = $\boxed{4x^{10}}$

3. $\sqrt[4]{81x^{20}}$ = $\boxed{3x^{5}}$

4. $-\sqrt{225x^{48}}$ = $\boxed{-15x^{24}}$

5. $\sqrt[3]{-125x^{57}}$ = $\boxed{-5x^{19}}$

6. $\sqrt[4]{-256x^{36}}$: $\boxed{\text{DNE in real numbers}}$

Simplify. Assume all variables are nonnegative.

1. $\sqrt{225x^{52}}$

$15x^{26}$

2. $\sqrt[3]{125x^{42}}$

$5x^{14}$

3. $\sqrt[4]{256x^{140}}$

$4x^{35}$

4. $-\sqrt{196x^{94}}$

$-14x^{47}$

5. $\sqrt[3]{-64x^{159}}$

$-4x^{53}$

6. $\sqrt[4]{-81x^{244}}$

DNE in
real numbers

Simplify. Assume all variables are nonnegative.

1. $\sqrt{-80x^{21}}$: $\boxed{\text{DNE in real numbers}}$

2. $\sqrt[3]{81x^{55}} = \sqrt[3]{27x^{54}} \; \sqrt[3]{3x}$

$= \boxed{3x^{18} \sqrt[3]{3x}}$

3. $\sqrt[4]{32x^{90}} = \sqrt[4]{16x^{88}} \; \sqrt[4]{2x^2}$

$= \boxed{2x^{22} \sqrt[4]{2x^2}}$

4. $\sqrt{63x^{25}} = \sqrt{9x^{24}}\sqrt{7x}$

$= \boxed{3x^{12}\sqrt{7x}}$

5. $-\sqrt[3]{-40x^{53}} = -\sqrt[3]{-8x^{51}}\sqrt[3]{5x^2}$

$= -(-2x^{17})\sqrt[3]{5x^2}$

$= \boxed{2x^{17}\sqrt[3]{5x^2}}$

6. $-\sqrt[4]{625x^{75}} = -\sqrt[4]{625x^{72}}\sqrt[4]{x^3}$

$= \boxed{-5x^{18}\sqrt[4]{x^3}}$

Simplify. Assume all variables are nonnegative.

1. $\sqrt{-63x^{59}}$

DNE in
real numbers

2. $\sqrt[3]{40x^{74}}$

$2x^{24}\sqrt[3]{5x^2}$

3. $\sqrt[4]{625x^{107}}$

$5x^{26}\sqrt[4]{x^3}$

4. $\sqrt{96x^{81}}$

$4x^{40}\sqrt{6x}$

5. $-\sqrt[3]{-81x^{94}}$

$3x^{31}\sqrt[3]{3x}$

6. $-\sqrt[4]{32x^{70}}$

$-2x^{17}\sqrt[4]{2x^2}$

Simplify. Assume all variables are nonnegative.

1. $-7\sqrt{360} + 5\sqrt{40} - 3\sqrt{490}$

$$= -7\sqrt{36}\sqrt{10} + 5\sqrt{4}\sqrt{10} - 3\sqrt{49}\sqrt{10}$$

$$= -7(6)\sqrt{10} + 5(2)\sqrt{10} - 3(7)\sqrt{10}$$

$$= -42\sqrt{10} + 10\sqrt{10} - 21\sqrt{10}$$

$$= \boxed{-53\sqrt{10}}$$

2. $3\sqrt[3]{16} - \sqrt[3]{128} + 4\sqrt[3]{54}$

$$= 3\sqrt[3]{8}\sqrt[3]{2} - \sqrt[3]{64}\sqrt[3]{2} + 4\sqrt[3]{27}\sqrt[3]{2}$$

$$= 3(2)\sqrt[3]{2} - 4\sqrt[3]{2} + 4(3)\sqrt[3]{2}$$

$$= 6\sqrt[3]{2} - 4\sqrt[3]{2} + 12\sqrt[3]{2}$$

$$= \boxed{14\sqrt[3]{2}}$$

3. $x\sqrt{54x^3} + 2\sqrt{24x^5}$

$$= x\sqrt{9x^2}\sqrt{6x} + 2\sqrt{4x^4}\sqrt{6x}$$

$$= x(3x)\sqrt{6x} + 2(2x^2)\sqrt{6x}$$

$$= 3x^2\sqrt{6x} + 4x^2\sqrt{6x}$$

$$= \boxed{7x^2\sqrt{6x}}$$

4. $\sqrt[3]{-4x^{11}} - x\sqrt[3]{500x^8}$

$= \sqrt[3]{-x^9}\ \sqrt[3]{4x^2} - x\ \sqrt[3]{125x^6}\ \sqrt[3]{4x^2}$

$= -x^3\ \sqrt[3]{4x^2} - x(5x^2)\sqrt[3]{4x^2}$

$= -x^3\ \sqrt[3]{4x^2} - 5x^3\ \sqrt[3]{4x^2}$

$= \boxed{-6x^3\ \sqrt[3]{4x^2}}$

5. $3\sqrt{75x^9} - 11x^4\sqrt{12x} + x\sqrt{48x^7}$

$= 3\sqrt{25x^8}\sqrt{3x} - 11x^4\sqrt{4}\ \sqrt{3x} + x\sqrt{16x^6}\sqrt{3x}$

$= 3(5x^4)\sqrt{3x} - 11x^4(2)\sqrt{3x} + x(4x^3)\sqrt{3x}$

$= 15x^4\sqrt{3x} - 22x^4\sqrt{3x} + 4x^4\sqrt{3x}$

$= \boxed{-3x^4\sqrt{3x}}$

6. $5\sqrt[3]{216x^7} - 3x\sqrt[3]{-8x^4} + 2x^2\sqrt[3]{64x}$

$= 5\sqrt[3]{216x^6}\ \sqrt[3]{x} - 3x\ \sqrt[3]{-8x^3}\ \sqrt[3]{x} + 2x^2\ \sqrt[3]{64}\ \sqrt[3]{x}$

$= 5(6x^2)\sqrt[3]{x} - 3x(-2x)\sqrt[3]{x} + 2x^2(4)\sqrt[3]{x}$

$= 30x^2\ \sqrt[3]{x} + 6x^2\ \sqrt[3]{x} + 8x^2\ \sqrt[3]{x}$

$= \boxed{44x^2\ \sqrt[3]{x}}$

Simplify. Assume all variables are nonnegative.

1. $-9\sqrt{600} + 4\sqrt{150} - 5\sqrt{486}$

$-115\sqrt{6}$

2. $3\sqrt[3]{256} - \sqrt[3]{108} + 2\sqrt[3]{32}$

$13\sqrt[3]{4}$

3. $2\sqrt{48x^9} + x^2\sqrt{363x^5}$

$19x^4\sqrt{3x}$

4. $\sqrt[3]{-2x^{17}} - x\sqrt[3]{250x^{14}}$

$-6x^5 \sqrt[3]{2x^2}$

5. $15\sqrt{490x^7} - 10x^3\sqrt{1960x} + x\sqrt{2250x^5}$

$-20x^3 \sqrt{10x}$

6. $5\sqrt[3]{27x^7} - x\sqrt[3]{-216x^4} + 4x^2\sqrt[3]{-x}$

$17x^2 \sqrt[3]{x}$

Multiply. Assume all variables are nonnegative.

1. $\sqrt{6x^9}\sqrt{8x^{12}} = \sqrt{48x^{21}}$

$$= \sqrt{16x^{20}}\sqrt{3x}$$

$$= \boxed{4x^{10}\sqrt{3x}}$$

2. $\sqrt[3]{20x^{14}}\sqrt[3]{4x^{21}} = \sqrt[3]{80x^{35}}$

$$= \sqrt[3]{8x^{33}}\sqrt[3]{10x^2}$$

$$= \boxed{2x^{11}\sqrt[3]{10x^2}}$$

3. $5x^3\sqrt{2x^7}\left(3x^4\sqrt{6x^3} - 2x\sqrt{8x^6}\right) = 15x^7\sqrt{12x^{10}} - 10x^4\sqrt{16x^{13}}$

$$= 15x^7\sqrt{4x^{10}}\sqrt{3} - 10x^4\sqrt{16x^{12}}\sqrt{x}$$

$$= 15x^7(2x^5)\sqrt{3} - 10x^4(4x^6)\sqrt{x}$$

$$= \boxed{30x^{12}\sqrt{3} - 40x^{10}\sqrt{x}}$$

4. $-2x\sqrt[3]{4x^2}\left(7x^3\sqrt[3]{4x^2} + 8x^5\sqrt[3]{5x}\right) = -14x^4\sqrt[3]{16x^4} - 16x^6\sqrt[3]{20x^3}$

$$= -14x^4\sqrt[3]{8x^3}\sqrt[3]{2x} - 16x^6\sqrt[3]{x^3}\sqrt[3]{20}$$

$$= -14x^4(2x)\sqrt[3]{2x} - 16x^6(x)\sqrt[3]{20}$$

$$= \boxed{-28x^5\sqrt[3]{2x} - 16x^7\sqrt[3]{20}}$$

5. $(8\sqrt{x} - 9)(7\sqrt{x} - 10) = 56\sqrt{x^2} - 80\sqrt{x} - 63\sqrt{x} + 90$

$$= \boxed{56x - 143\sqrt{x} + 90}$$

6. $(5\sqrt{x} + 13)(5\sqrt{x} - 13) = 25\sqrt{x^2} - 65\sqrt{x} + 65\sqrt{x} - 169$

$$= \boxed{25x - 169}$$

Multiply. Assume all variables are nonnegative.

1. $\sqrt{8x^{17}}\sqrt{12x^{84}}$

$4x^{50}\sqrt{6x}$

2. $\sqrt[3]{4x^{25}}\sqrt[3]{10x^{31}}$

$2x^{18}\sqrt[3]{5x^2}$

3. $3x^5\sqrt{7x^9}\left(4x^3\sqrt{8x^7} - 9x\sqrt{7x^2}\right)$

$24x^{16}\sqrt{14} - 189x^{11}\sqrt{x}$

4. $-4x\sqrt[3]{2x^4}\left(5x^2\sqrt[3]{2x^4}+3x^3\sqrt[3]{9x^2}\right)$

$-20x^5\sqrt[3]{4x^2}-12x^6\sqrt[3]{18}$

5. $\left(9\sqrt{x}-8\right)\left(10\sqrt{x}-7\right)$

$90x-143\sqrt{x}+56$

6. $\left(15\sqrt{x}+8\right)\left(15\sqrt{x}-8\right)$

$225x-64$

Rationalize the denominator. Assume all variables are nonnegative.

1. $\sqrt{\dfrac{25}{30}} = \sqrt{\dfrac{5}{6}} = \dfrac{\sqrt{5}}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{\sqrt{30}}{\sqrt{36}} = \boxed{\dfrac{\sqrt{30}}{6}}$

2. $\dfrac{6x^2}{\sqrt[3]{9x}} \cdot \dfrac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \dfrac{6x^2\sqrt[3]{3x^2}}{\sqrt[3]{27x^3}} = \dfrac{\overset{2}{\cancel{6}}x^{\overset{1}{\cancel{2}}}\sqrt[3]{3x^2}}{\cancel{3}\cancel{x}} = \boxed{2x\sqrt[3]{3x^2}}$

3. $\dfrac{20x}{\sqrt{5x^3}} \cdot \dfrac{\sqrt{5x}}{\sqrt{5x}} = \dfrac{20x\sqrt{5x}}{\sqrt{25x^4}} = \dfrac{\overset{4}{\cancel{20}}\cancel{x}\sqrt{5x}}{\cancel{5}x^{\cancel{4}}} = \boxed{\dfrac{4\sqrt{5x}}{x}}$

Rationalize the numerator. Assume all variables are nonnegative.

4. $\sqrt[3]{\dfrac{32}{40}} = \sqrt[3]{\dfrac{4}{5}} = \dfrac{\sqrt[3]{4}}{\sqrt[3]{5}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{10}} = \boxed{\dfrac{2}{\sqrt[3]{10}}}$

5. $\dfrac{\sqrt{54x^6}}{\sqrt{16x^5}} = \dfrac{\sqrt{27x}}{\sqrt{8}} \cdot \dfrac{\sqrt{3x}}{\sqrt{3x}} = \dfrac{\sqrt{81x^2}}{\sqrt{24x}} = \dfrac{9x}{\sqrt{24x}} = \dfrac{9x}{\sqrt{4}\,\sqrt{6x}} = \boxed{\dfrac{9x}{2\sqrt{6x}}}$

6. $\dfrac{\sqrt[3]{45x^{10}}}{\sqrt[3]{72x^8}} = \dfrac{\sqrt[3]{5x^2}}{\sqrt[3]{8}} = \dfrac{\sqrt[3]{5x^2}}{2} \cdot \dfrac{\sqrt[3]{25x}}{\sqrt[3]{25x}} = \dfrac{\sqrt[3]{125x^3}}{2\sqrt[3]{25x}} = \boxed{\dfrac{5x}{2\sqrt[3]{25x}}}$

Rationalize the denominator. Assume all variables are nonnegative.

1. $\sqrt{\dfrac{30}{35}}$

$\dfrac{\sqrt{42}}{7}$

2. $\dfrac{15x^2}{\sqrt[3]{5x}}$

$3x\sqrt[3]{25x^2}$

3. $\dfrac{21x^3}{\sqrt{3x^7}}$

$\dfrac{7\sqrt{3x}}{x}$

Rationalize the numerator. Assume all variables are nonnegative.

4. $\sqrt[3]{\dfrac{180}{25}}$

$\dfrac{6}{\sqrt[3]{30}}$

5. $\dfrac{\sqrt{24x^9}}{\sqrt{135x^8}}$

$\dfrac{4x}{3\sqrt{10x}}$

6. $\dfrac{\sqrt[3]{18x^{15}}}{\sqrt[3]{250x^{13}}}$

$\dfrac{3x}{5\sqrt[3]{3x}}$

Solve.

1. $\sqrt{3x-5} = 2$

$$\left(\sqrt{3x-5}\right)^2 = 2^2$$

$$3x - 5 = 4$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{9}{3}$$

$$\boxed{x = 3}$$

Check: $\sqrt{3(3)-5} \overset{?}{=} 2$

$$\sqrt{9-5} \overset{?}{=} 2$$

$$\sqrt{4} \overset{?}{=} 2$$

$$2 = 2 \checkmark$$

2. $\sqrt[3]{\frac{3}{4}x + 29} = 3$

$$\left(\sqrt[3]{\frac{3}{4}x + 29}\right)^3 = 3^3$$

$$\cancel{4}\left(\frac{3}{\cancel{4}}x + 29\right) = (27)$$

$$3x + 116 = 108$$

$$\frac{\cancel{3}x}{\cancel{3}} = -\frac{8}{3}$$

$$\boxed{x = -\frac{8}{3}}$$

3. $\sqrt[4]{7x-6} = -1$

$$\left(\sqrt[4]{7x-6}\right)^4 = (-1)^4$$

$$7x - 6 = 1$$

$$\frac{\cancel{7}x}{\cancel{7}} = \frac{7}{7}$$

$$\cancel{x = 1}$$

$$\boxed{\text{No solution}}$$

Check: $\sqrt[4]{7(1)-6} \overset{?}{=} -1$

$$\sqrt[4]{7-6} \overset{?}{=} -1$$

$$\sqrt[4]{1} \overset{?}{=} -1$$

$$1 = -1 \quad \times$$

4. $\sqrt{2x^2 + 13x + 45} = 5$

$(\sqrt{2x^2+13x+45})^2 = 5^2$

$2x^2 + 13x + 45 = 25$

$2x^2 + 13x + 20 = 0$ $\qquad \frac{40}{8,5}$

$2x^2 + 8x + 5x + 20 = 0$

$2x(x+4) + 5(x+4) = 0$

$(x+4)(2x+5) = 0$

$\boxed{x = -4, \ -5/2}$

Check $x = -4$

$\sqrt{2(-4)^2 + 13(-4) + 45} \overset{?}{=} 5$

$\sqrt{2(16) + 13(-4) + 45} \overset{?}{=} 5$

$\sqrt{32 - 52 + 45} \overset{?}{=} 5$

$\sqrt{25} \overset{?}{=} 5$

$5 = 5 \checkmark$

Check $x = -\frac{5}{2}$

$\sqrt{2(-5/2)^2 + 13(-5/2) + 45} \overset{?}{=} 5$

$\sqrt{2(\frac{25}{4}) + 13(-5/2) + 45} \overset{?}{=} 5$

$\sqrt{\frac{25}{2} - \frac{65}{2} + 45} \overset{?}{=} 5$

$\sqrt{-\frac{40}{2} + 45} \overset{?}{=} 5$

$\sqrt{-20 + 45} \overset{?}{=} 5$

$\sqrt{25} \overset{?}{=} 5$

$5 = 5 \checkmark$

5. $\sqrt[3]{25x^2 - 80} = -4$

$(\sqrt[3]{25x^2 - 80})^3 = (-4)^3$

$25x^2 - 80 = -64$

$25x^2 - 16 = 0$

$(5x+4)(5x-4) = 0$

$\boxed{x = -4/5, \ 4/5}$

6. $\sqrt[4]{\frac{1}{12}x^2 - \frac{5}{6}x + 18} = 2$

$(\sqrt[4]{\frac{1}{12}x^2 - \frac{5}{6}x + 18})^4 = (2)^4$

$12(\frac{1}{12}x^2 - \frac{5}{6}x + 18) = (16)12$

$x^2 - 10x + 216 = 192$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$\boxed{x = 4, 6}$

Check $x = 4$

$\sqrt[4]{\frac{1}{12}(4)^2 - \frac{5}{6}(4) + 18} \overset{?}{=} 2$

$\sqrt[4]{\frac{1}{12}(16) - \frac{5}{6}(4) + 18} \overset{?}{=} 2$

$\sqrt[4]{4/3 - 10/3 + 18} \overset{?}{=} 2$

$\sqrt[4]{-6/3 + 18} \overset{?}{=} 2$

$\sqrt[4]{-2 + 18} \overset{?}{=} 2$

$\sqrt[4]{16} \overset{?}{=} 2$

$2 = 2 \checkmark$

Check $x = 6$

$\sqrt[4]{\frac{1}{12}(6)^2 - \frac{5}{6}(6) + 18} = 2$

$\sqrt[4]{\frac{1}{12}(36) - \frac{5}{6}(6) + 18} \overset{?}{=} 2$

$\sqrt[4]{3 - 5 + 18} \overset{?}{=} 2$

$\sqrt[4]{16} \overset{?}{=} 2$

$2 = 2 \checkmark$

Solve.

1. $\sqrt{1 - 18x} = 3$

$x = -\frac{4}{9}$

2. $\sqrt[3]{\frac{8}{5}x - 129} = -5$

$x = \frac{5}{2}$

3. $\sqrt[4]{3x + 1} = -2$

No solution

4. $\sqrt{2x^2 + 13x + 24} = 3$

$x = -\frac{3}{2}, -5$

5. $\sqrt[3]{64x^2 - 225} = -6$

$x = -\frac{3}{8}, \frac{3}{8}$

6. $\sqrt[4]{\frac{1}{16}x^2 - \frac{5}{8}x + 2} = 1$

$x = 2, 8$

Solve by the square root property.

1. $\left(\frac{1}{4}(2x+1)^2 + 5\right) = (20)$

$(2x+1)^2 + 20 = 80$

$(2x+1)^2 = 60$

$\sqrt{(2x+1)^2} = \pm\sqrt{60}$

$2x+1 = \pm 2\sqrt{15}$

$2x = -1 \pm 2\sqrt{15}$

$$\boxed{x = \frac{-1 \pm 2\sqrt{15}}{2}}$$

$\sqrt{60} = \sqrt{4}\sqrt{15}$
$= 2\sqrt{15}$

2. $4(7x-5)^2 - 187 = 101$

$\frac{4(7x-5)^2}{4} = \frac{288}{4}$

$(7x-5)^2 = 72$

$\sqrt{(7x-5)^2} = \pm\sqrt{72}$

$7x - 5 = \pm 6\sqrt{2}$

$7x = 5 \pm 6\sqrt{2} \implies \boxed{x = \frac{5 \pm 6\sqrt{2}}{7}}$

$\sqrt{72} = \sqrt{36}\sqrt{2}$
$= 6\sqrt{2}$

3. $\left(\frac{5}{6}(3x+4)^2 - 1\right) = \left(\frac{3}{2}\right)$

$5(3x+4)^2 - 6 = 9$

$\frac{5(3x+4)^2}{5} = \frac{15}{5}$

$(3x+4)^2 = 3$

$\sqrt{(3x+4)^2} = \pm\sqrt{3}$

$3x+4 = \pm\sqrt{3}$

$3x = -4 \pm\sqrt{3} \implies \boxed{x = \frac{-4 \pm\sqrt{3}}{3}}$

4. $2(5x-1)^2 + 9 = 117$

$$\frac{2(5x-1)^2}{2} = \frac{108}{2}$$

$$(5x-1)^2 = 54$$

$$\sqrt{(5x-1)^2} = \pm\sqrt{54}$$

$$5x-1 = \pm 3\sqrt{6}$$

$$\frac{5x}{5} = \frac{1 \pm 3\sqrt{6}}{5}$$

$$\boxed{x = \frac{1 \pm 3\sqrt{6}}{5}}$$

$$\sqrt{54} = \sqrt{9}\sqrt{6}$$
$$= 3\sqrt{6}$$

5. $\left(\frac{7}{3}(4x+9)^2 - 150\right)^3 = \left(\frac{100}{3}\right)^3$

$$7(4x+9)^2 - 450 = -100$$

$$\frac{7(4x+9)^2}{7} = \frac{350}{7}$$

$$(4x+9)^2 = 50$$

$$\sqrt{(4x+9)^2} = \pm\sqrt{50}$$

$$4x+9 = \pm 5\sqrt{2}$$

$$\frac{4x}{4} = \frac{-9 \pm 5\sqrt{2}}{4} \implies \boxed{x = \frac{-9 \pm 5\sqrt{2}}{4}}$$

$$\sqrt{50} = \sqrt{25}\sqrt{2}$$
$$= 5\sqrt{2}$$

6. $-3(6x-7)^2 + 125 = -10$

$$\frac{-3(6x-7)^2}{-3} = \frac{-135}{-3}$$

$$(6x-7)^2 = 45$$

$$\sqrt{(6x-7)^2} = \pm\sqrt{45}$$

$$6x-7 = \pm 3\sqrt{5}$$

$$\frac{6x}{6} = \frac{7 \pm 3\sqrt{5}}{6}$$

$$\boxed{x = \frac{7 \pm 3\sqrt{5}}{6}}$$

$$\sqrt{45} = \sqrt{9}\sqrt{5}$$
$$3\sqrt{5}$$

Solve by the square root property.

1. $\frac{1}{5}(7x+2)^2 + 1 = 10$
$$x = \frac{-2 \pm 3\sqrt{5}}{7}$$

2. $9(3x-8)^2 - 271 = 179$
$$x = \frac{8 \pm 5\sqrt{2}}{3}$$

3. $\frac{5}{6}(9x+1)^2 - \frac{10}{3} = \frac{5}{2}$
$$x = \frac{-1 \pm \sqrt{7}}{9}$$

4. $7(4x - 3)^2 + 15 = 435$

$$x = \frac{3 \pm 2\sqrt{15}}{4}$$

5. $\frac{9}{4}(2x + 5)^2 - 55 = -\frac{29}{2}$

$$x = \frac{-5 \pm 3\sqrt{2}}{2}$$

6. $-4(8x - 7)^2 + 211 = -5$

$$x = \frac{7 \pm 3\sqrt{6}}{8}$$

Solve by the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $3x^2 + 3x - 1 = 0$

$a = 3 \quad b = 3 \quad c = -1$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(3)(-1)}}{2(3)} \overset{9+12}{=} \boxed{\frac{-3 \pm \sqrt{21}}{6}}$$

2. $\overset{4}{\overbrace{\left(\tfrac{3}{4}x^2\right)}} = \overset{4}{\overbrace{\left(1 + \tfrac{1}{2}x\right)}}$

$3x^2 = 4 + 2x$

$3x^2 - 2x - 4 = 0$

$a = 3 \quad b = -2 \quad c = -4$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)} \overset{4+48}{=} \frac{2 \pm \sqrt{52}}{6} \overset{\sqrt{4}\sqrt{13}}{\underset{2\sqrt{13}}{=}} \frac{2 \pm 2\sqrt{13}}{6} = \boxed{\frac{1 \pm \sqrt{13}}{3}}$$

3. $(2x + 7)^2 - 30x = 2x(x - 5) + 44$

$(2x+7)(2x+7)$

$4x^2 + 14x + 14x + 49 - 30x = 2x^2 - 10x + 44$

$4x^2 - 2x + 49 = 2x^2 - 10x + 44$

$2x^2 + 8x + 5 = 0$

$a = 2 \quad b = 8 \quad c = 5$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(5)}}{2(2)} \overset{64-40}{=} \frac{-8 \pm \sqrt{24}}{4} \overset{\sqrt{4}\sqrt{6}}{\underset{2\sqrt{6}}{=}} \frac{-8 \pm 2\sqrt{6}}{4} = \boxed{\frac{-4 \pm \sqrt{6}}{2}}$$

4. $4x^2 - 6x + 1 = 0$

$a = 4 \quad b = -6 \quad c = 1$

$$x = \frac{-(-6) \pm \sqrt{\overset{36-16}{(-6)^2 - 4(4)(1)}}}{2(4)} = \frac{6 \pm \overset{\overset{2\sqrt{5}}{\sqrt{4}\sqrt{5}}}{\sqrt{20}}}{8} = \frac{6 \pm 2\sqrt{5}}{8}$$

$$= \boxed{\frac{3 \pm \sqrt{5}}{4}}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

5. $\overset{100}{\overbrace{\left(\tfrac{1}{4}x^2 + \tfrac{1}{10}x\right.}} = \overset{100}{\overbrace{\left.\tfrac{1}{100}\right)}}$

$25x^2 + 10x = 1$

$25x^2 + 10x - 1 = 0$

$a = 25 \quad b = 10 \quad c = -1$

$$x = \frac{-10 \pm \sqrt{\overset{100+100}{10^2 - 4(25)(-1)}}}{2(25)} = \frac{-10 \pm \overset{\overset{10\sqrt{2}}{\sqrt{100}\sqrt{2}}}{\sqrt{200}}}{50} = \frac{-10 \pm 10\sqrt{2}}{50} = \boxed{\frac{-1 \pm \sqrt{2}}{5}}$$

6. $(4x - 5)(2x + 3) - 16x = 7x(x - 1) - 23$

$8x^2 + 12x - 10x - 15 - 16x = 7x^2 - 7x - 23$

$8x^2 - 14x - 15 = 7x^2 - 7x - 23$

$x^2 - 7x + 8 = 0$

$a = 1 \quad b = -7 \quad c = 8$

$$x = \frac{-(-7) \pm \sqrt{\overset{49-32}{(-7)^2 - 4(1)(8)}}}{2(1)} = \boxed{\frac{7 \pm \sqrt{17}}{2}}$$

Solve by the Quadratic Formula.

1. $5x^2 + 5x + 1 = 0$

$$x = \frac{-5 \pm \sqrt{5}}{10}$$

2. $\frac{3}{4}x^2 = x + \frac{1}{2}$

$$x = \frac{2 \pm \sqrt{10}}{3}$$

3. $(4x + 9)^2 - 71x = x(7x - 5) + 82$

$$x = \frac{-1 \pm \sqrt{2}}{3}$$

4. $2x^2 - 6x + 3 = 0$

$$x = \frac{3 \pm \sqrt{3}}{2}$$

5. $\frac{1}{5}x^2 + \frac{1}{6}x = \frac{1}{15}$

$$x = \frac{-5 \pm \sqrt{73}}{12}$$

6. $(5x - 2)(3x + 7) - 44x = 7x(2x - 1) - 10$

$$x = 4 \pm 2\sqrt{5}$$